1. Suppose A and B divide a cake consisting of 4 homogeneous components, C, S, P, and W. Their preferences are given below.

A's Preferences

\[
\begin{array}{ccc}
6a & 3a & 5a \\
C & S & W \\
\end{array}
\]

B's Preferences

\[
\begin{array}{ccc}
2b & b & a \\
C & S & W \\
\end{array}
\]

Fill in the table below with their valuations of the components:

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>S</th>
<th>P</th>
<th>W</th>
<th>Reduced Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>6/15</td>
<td>3/15</td>
<td>1/15</td>
<td>5/15</td>
<td>[\frac{2}{15}, \frac{1}{15}, \frac{1}{15}, \frac{1}{15}]</td>
</tr>
<tr>
<td>B</td>
<td>2/15</td>
<td>1/15</td>
<td>1/15</td>
<td>1/15</td>
<td>[\frac{2}{15}, \frac{1}{15}, \frac{1}{15}, \frac{1}{15}]</td>
</tr>
</tbody>
</table>

Using this information, please answer the following questions.

(a) Compute the A-to-B valuation ratios for each component.

\[
\begin{array}{ccc|cc}
| & C | S | P | W \\
\hline
A-to-B valuation ratio & 1 & 1 & 1/3 & 5/3 \\
\end{array}
\]

(b) Draw the threshold diagram (be sure to label your dots).

(c) Draw the threshold division for the ratio \( r = \frac{4}{3} \) and answer the following questions.

i. List the fraction of each component that A and B each receive in this division.

\[
\begin{align*}
A : & & \circ C + \circ S + \circ P + \bigcirc W \\
B : & & \bigcirc C + \bigcirc S + \bigcirc P + \circ W
\end{align*}
\]

ii. Find each person's share for this division.

\[
A's \text{ share} = \frac{1}{3}, \quad B's \text{ share} = \frac{4}{15}
\]
iii. Is the division: (please fill in the blank with 'Y' for Yes and 'N' for No)

\[ \frac{N}{N} \text{ fair} \]
\[ \frac{N}{N} \text{ equitable} \]
\[ Y \text{ Pareto optimal} \]

iv. If the division is not equitable, should we move the division line to the right or left if we want an equitable division? (think about which person’s share needs to be increased and which needs to be decrease for equitability to be achieved).

to the right \(-\) gives more to A

d) Draw the threshold division for the ratio \( r = \frac{2}{3} \) and answer the following questions.

\[ \begin{array}{c}
\circ P \\
C/3 \\
\circ S \\
\circ P \\
\circ W \\
\end{array} \]

i. List the fraction of each component that A and B each receive in this division.

\[ \begin{align*}
A : & \quad \underline{1} C + \underline{1} S + \underline{0} P + \underline{1} W \\
B : & \quad \underline{0} C + \underline{0} S + \underline{1} P + \underline{0} W
\end{align*} \]

ii. Find each person’s share for this division.

\[ A's \text{ share} = \frac{14}{15} \]
\[ B's \text{ share} = \frac{1}{15} \]

iii. Is the division: (please fill in the blank with 'Y' for Yes and 'N' for No)

\[ \frac{N}{N} \text{ fair} \]
\[ \frac{N}{N} \text{ equitable} \]
\[ Y \text{ Pareto optimal} \]

iv. If the division is not equitable, should we move the division line to the right or left if we want an equitable division? (think about which person’s share needs to be increased and which needs to be decrease for equitability to be achieved).

to the left \(-\) gives more to B

e) Would the equal division be a threshold division in this example?

Circle One: Yes \( \bigcirc \) No

\[ \text{The equal division is a threshold division if and only if all } A \text{-to-} B \text{ value ratios are the same.} \]
\[ \text{if and only if all } k \text{-to-} B \text{ value ratios equal } 1 \]
\[ \text{if and only if } A \text{ and } B \text{ have exact same values for the components.} \]