1. Suppose two people, Peter ($P$) and Dina ($D$), wish to share the following cake which is $\frac{1}{3}$ Chocolate (C), $\frac{1}{3}$ Blueberry (B) and $\frac{1}{3}$ Walnut:

Suppose their preferences are as follows:

P’s Preferences  

\[
\begin{array}{c}
C & B & W \\
5p & 0 & 2p \\
\end{array}
\]

D’s Preferences  

\[
\begin{array}{c}
C & B & W \\
d & 2d & 3d \\
\end{array}
\]

Fill in the following table with their valuations of the different components:

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>B</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$D$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Suppose they decide to use **I cut, you choose** to divide the cake where $D$ is the cutter and $P$ is the chooser. Justify all your answers below.

(a) Verify that $D$ could cut as follows.

(b) What does $P$ think each slice is worth?
(c) List the division that could result from using I cut, you choose with the cut given above by listing the slice each of P and D receives in the blanks below.

\[
\begin{align*}
P & : \\
D & : 
\end{align*}
\]

(d) Is the division envy-free?

(e) Is the division equitable?

(f) Is the division Pareto optimal?

2. Suppose 2 housemates Adam (A) and Bob (B), who are moving out, wish to share a DVD collection of 12 DVDs consisting of 3 types:

- 2 Romance DVDs (R)
- 4 Horror DVDs (H)
- 6 Comedy DVDs (C)

We will represent the DVDs in the following diagram where one small square represents 1 DVD (all small squares are identical in area):

```
H R R
H C C
H C C
H C C
```

A and B’s preferences for the different types of DVDs are as follows:

- A likes all 3 types of DVDs equally.
- B likes Romance and Comedy DVDs equally but likes Horror twice as much as he likes either of the others.
(a) Fill in the charts below with A and B’s preferences given that:

\[ a = \text{the amount that } A \text{ values 1 Comedy DVD} \]
\[ b = \text{the amount that } B \text{ values 1 Comedy DVD} \]

<table>
<thead>
<tr>
<th>A’s preferences:</th>
<th></th>
<th></th>
<th></th>
<th>total = ____ a</th>
</tr>
</thead>
<tbody>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B’s preferences:</th>
<th></th>
<th></th>
<th></th>
<th>total = ____ b</th>
</tr>
</thead>
<tbody>
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<td></td>
</tr>
</tbody>
</table>

(b) Fill in the following table with their valuations of the different components:

<table>
<thead>
<tr>
<th></th>
<th>H</th>
<th>R</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) Suppose A and B want to share the DVDs using the method of I cut, you choose where B cuts and A chooses. Answer the following questions.

i. In which of the following ways might B cut? Circle all that apply.

ii. Suppose B cuts as follows:

\[
S_1 \quad S_2
\]

\[
S_1
\]

\[
S_2
\]
iii. What does A think each slice is worth?

<table>
<thead>
<tr>
<th></th>
<th>(S_1)</th>
<th>(S_2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

iv. List the division that could result from using I cut, you choose with the cut given above by listing the slice each of \(A\) and \(B\) receives in the blanks below. This will be your original division.

\[
\begin{align*}
A &: \_\_\_ \quad (A's \ share) = \_\_\_ \\
B &: \_\_\_ \quad (B's \ share) = \_\_\_
\end{align*}
\]

v. Is the division envy-free?

vi. Is the division equitable?

vii. Consider the alternative division below.

\[
\begin{array}{|c|}
\hline
\text{S}_1' \\
\text{H} \frac{R}{H} \frac{R}{C} \frac{C}{C} \\
\text{H} \frac{C}{C} \\
\text{H} \\
\hline
\end{array}
\begin{array}{|c|}
\hline
\text{S}_2' \\
\text{H} \frac{R}{H} \frac{R}{C} \frac{C}{C} \\
\text{H} \frac{C}{C} \\
\text{H} \\
\hline
\end{array}
\]

\(A\) gets \(S_2'\) and \(B\) gets \(S_1'\)

A. Fill in the following table with their valuations of the slices in the alternative division above:

<table>
<thead>
<tr>
<th></th>
<th>(S_1')</th>
<th>(S_2')</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(B)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B. Identify each person’s share in this alternative division.

\[
\begin{align*}
(A's \ share) &= \_\_\_ \\
(B's \ share) &= \_\_\_
\end{align*}
\]

C. Is this alternative division an objective improvement over the original division?

Circle One: Yes No

Please Explain.