Math 70	TUFTS UNIVERSITY	May 9, 2016, 8:30-10:30 A.M.
Linear Algebra	Final Exam	Department of Mathematics

Instructions: No notes or books are allowed. All calculators, cell phones, or other electronic devices **must** be turned off and put away during the exam. Unless otherwise stated, you **must show all work** to receive full credit. *You are required to sign the last page of your exam. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

Problem	Point Value	Points
1	10	
2	2	
3	6	
4	8	
5	10	
6	8	
7	10	
8	8	
9	8	
10	6	
11	8	
12	8	
13	8	
	100	

1. (10 points) For each question, indicate your answer by shading the appropriate box. No partial credit.

(a) $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$ [T] [F]

to be a subspace
a set needs to be a subset
and $\mathbb{R}^2 \not\subset \mathbb{R}^3$

(b) $\mathbb{P}_2$ is a subspace of $\mathbb{P}_3$ ($\mathbb{P}_n$ is the set of polynomials of degree less than or equal to n). [T] [F]

yes as every poly of deg ≤ 2
is of deg ≤ 3

(c) Is it possible to have a linear transformation $T : \mathbb{R}^n \to \mathbb{R}^n$ with the property that $T(u) = T(v)$ for some pair of distinct vectors u and v in $\mathbb{R}^n$ *and* that T is onto $\mathbb{R}^n$? [YES] [NO]

if $T: \mathbb{R}^n \to \mathbb{R}^n$ is Linear then T
is one-to-one iff T is onto

(d) Every orthogonal set in $\mathbb{R}^n$ has at most n vectors in it. [T] [F]

no every orthogonal set of nonzero vectors
has $\leq n$ vectors as
orthogonal nonzero vectors are independent

(e) If the orthogonal projection of a vector v onto a subspace W equals v, then $\mathrm{v} \in W$. [T] [F]

The orthogonal projection
is in W

2. (2 points) Let V be a vector space. Consider the three sets
   i. S_1 is a linearly independent subset of V but it does not span V;
   ii. S_2 is a spanning set of V but it is not linearly independent, and
   iii. S_3 is a basis of V.

Order the sets from smallest to largest in the spaces below.

i ___ iii ___ ii ___

indep set $\leq$ dim V
basis $=$ dim V.
spanning set $\geq$ dim V and if dep $>$ dim V.

3. (6 points) Let A be an $n \times n$ matrix such that $\det(A^4) = 0$. Is A invertible? Justify your answer.

no as $\det A^4 = \det(A \cdot A \cdot A \cdot A)$

$= (\det A)(\det A)(\det A)(\det A)$

$= (\det A)^4$

if $(\det A)^4 = 0$ then $\det A = 0$. so A is not invertible

4. (8 points) Let $A = \begin{bmatrix} 4 & 1 \\ 3 & 6 \end{bmatrix}$.

(a) Find all eigenvalues of A.

$\det(A - \lambda I) = \det \begin{bmatrix} 4-\lambda & 1 \\ 3 & 6-\lambda \end{bmatrix}$

$= (4-\lambda)(6-\lambda) - 3 = \lambda^2 - 10\lambda + 21$

$= (\lambda - 7)(\lambda - 3)$

(b) Show that A is diagonalizable by finding an invertible matrix P and diagonal matrix D such that $A = PDP^{-1}$

find evects: $A - 7I = \begin{bmatrix} -3 & 1 \\ 3 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1/3 \\ 0 & 0 \end{bmatrix}$

if $\vec{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$ then $v_1 = \frac{v_2}{3}$ so if $v_2 = 3$, $v_1 = 1$

$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ evect for $\lambda = 7$

$\lambda = 3$ $A - 3I = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ $v_1 = -v_2$

$\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ evect for $\lambda = 3$

$P = \begin{bmatrix} 1 & -1 \\ 3 & 1 \end{bmatrix}$ $D = \begin{bmatrix} 7 & 0 \\ 0 & 3 \end{bmatrix}$

as A has an eigenvector basis for $\mathbb{R}^2$ (each e-space has dimension = multiplicity of the eval in the characteristic equn., A is diagonalizable)

5. (10 points) Suppose A is a 4×4 matrix *and assume $\lambda = 0$ is an eigenvalue of A.*

(a) Define what it means that $\lambda = 0$ is an eigenvalue of A.

for some $\bar{v} \in \mathbb{R}^4$ with $\bar{v} \neq 0$ $A\bar{v} = 0\bar{v}$

(b) Use the assumption that $\lambda = 0$ is an eigenvalue and the definition of linear dependence to prove that the columns of A are linearly dependent.

~~if~~ by above for some $\bar{v} \neq \bar{0}$ $A\bar{v} = 0\bar{v} = \bar{0}$

so as $A\bar{v} = \bar{0}$ has nontrivial solution

the columns of A are linearly dependent.

(c) The maximum rank of A (dimension of Col A) is 3.

If the columns of A are dependent then

by Theorem 4 on P208 as the vectors are dependent,

one is a linear comb of the others. By Thm 5

on p 210 some subset of is a basis. ∴ As there are 4

vectors that are dependent the rank ≤ 3

6. (8 points) Let $A = \begin{bmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{bmatrix}$. The characteristic polynomial of A is $p(\lambda) = (\lambda - 1)(\lambda + 3)^2$.

(a) Find a basis for the eigenspace corresponding to $\lambda = -3$.

$$A + 3I = \begin{bmatrix} 8 & 8 & 16 \\ 4 & 4 & 8 \\ -4 & -4 & -8 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$, if $\bar{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix}$

$v_1 = -v_2 - 2v_3$

basis: $v_2 = 1$, $v_3 = 0$ so $v_1 = -1$; $v_2 = 0$, $v_3 = 1$ so $v_1 = -2$

$$\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} \right\}$$

(b) Is A diagonalizable? Justify your answer.

Yes. As $\lambda = 1$ is an eval, there are eigenvectors for $\lambda = 1$ $\therefore$ The e-space for $\lambda = 1$ is at least one-dimentional (and at most 1 diml. because the mult. of the eval $\lambda = 1$ is one)

$\therefore$ there is an eigen vector basis of $\mathbb{R}^3$ (the total # of indep evects ~~are~~ is $3 = \dim \mathbb{R}^3$)

7. (10 points) Define the transformation $T : \mathbb{P}_2 \to \mathbb{M}_{2\times 2}$ by $T(a+bt+ct^2) = \begin{bmatrix} a+2b & b-c \\ 5c & 0 \end{bmatrix}$.
Then T is linear. You do not need to show this.

Let $\mathcal{B} = \{1, t, t^2\}$ and $\mathcal{C} = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$

be bases for $\mathbb{P}_2$ and $\mathbb{M}_{2\times 2}$, respectively. Find each of the following:

(a) $T(4t+5t^2)$ $= \begin{bmatrix} 8 & -1 \\ 25 & 0 \end{bmatrix}$

(b) The kernel of T. $T(a+bt+ct^2) = \begin{bmatrix} a+2b & b-c \\ 5c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

so $a+2b=0$, $b-c=0$, $5c=0$

$\begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ so the only soln is the trivial soln.

$\text{Ker } T = \{0\}$ (the 0 polynomial)

$\therefore T$ is one-to-one.

(c) The matrix for T relative to the bases $\mathcal{B}$ and $\mathcal{C}$. (Referred to as ${}_{\mathcal{C}}[T]_{\mathcal{B}}$ or ${}_{\mathcal{C}}M_{\mathcal{B}}$.)

$${}_C[T]_B = \left[\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}_C \; \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}_C \; \begin{bmatrix} 0 & -1 \\ 5 & 0 \end{bmatrix}_C \right]$$

$$= \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

because $\begin{bmatrix} a & b \\ c & d \end{bmatrix}_C = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

here we use

$${}_C[T]_B = \left[[T(1)]_C \; [T(t)]_C \; [T(t^2)]_C \right]$$

("the coordinates in the C basis of T of the B basis")

8. (8 points) Let $T : \mathbb{P}_2 \to W$ be a linear transformation.

Let $\mathcal{B} = \{1, t, t^2\}$ and $\mathcal{C} = \{e^x, \cos(x), \sin(x)\}$ be bases for $\mathbb{P}_2$ and W, respectively.

Let $M = {}_{\mathcal{C}}[T]_{\mathcal{B}} = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 1 \\ -1 & 1 & 3 \end{bmatrix}$ be the matrix of the transformation relative to the bases $\mathcal{B}$ and $\mathcal{C}$.

(a) Find $[4 - 3t + t^2]_{\mathcal{B}}$ $= \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix}$ as $4 - 3t + t^2 = 4 \cdot 1 + -3t + 1t^2$

(b) Find $T(4 - 3t + t^2)$. the only info we have is in terms of coordinates so we use:

$$[T\vec{v}]_C = {}_C[T]_B \, [\vec{v}]_B.$$

ie $$[T(4-3t+t^2)]_C = \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 1 \\ -1 & 1 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ -3 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \\ -4 \end{bmatrix}$$

so $T(4-3t+t^2) = 8e^x + -2\cos(x) + -4\sin(x)$

9. (8 points) Let $\mathbf{w}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{w}_2 = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$ and let $\mathbf{b} = \begin{bmatrix} 4 \\ 0 \\ 8 \end{bmatrix}$.

(a) Show that $\mathbf{w}_1$ and $\mathbf{w}_2$ are orthogonal.

Check $\bar{w}_1 \cdot \bar{w}_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix} = 2 - 4 + 2 = 0$ ✓

(b) Find the distance from $\mathbf{b}$ to $W = \text{Span}\{\mathbf{w}_1, \mathbf{w}_2\}$.

first find $\text{proj}_{\text{span}\{\bar{w}_1, \bar{w}_2\}} \bar{b} = \frac{\bar{b} \cdot \bar{w}_1}{\bar{w}_1 \cdot \bar{w}_1} \bar{w}_1 + \frac{\bar{b} \cdot \bar{w}_2}{\bar{w}_2 \cdot \bar{w}_2} \bar{w}_2$

$$= \frac{\begin{bmatrix} 4 \\ 0 \\ 8 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}}{3} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + \frac{\begin{bmatrix} 4 \\ 0 \\ 8 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}}{24} \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$$

$= 4$ $\quad$ $= 1$

$$= \begin{bmatrix} 4 \\ -4 \\ 4 \end{bmatrix} + \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix}$$

then $\bar{z} = \bar{b} - \text{proj}_{\text{span}\{\bar{w}_1, \bar{w}_2\}} \bar{b} \in W^{\perp}$ and

$\|z\| = \left\| \begin{bmatrix} 4 \\ 0 \\ 8 \end{bmatrix} - \begin{bmatrix} 6 \\ 0 \\ 6 \end{bmatrix} \right\| = 2\sqrt{2}$

is the distance from $\bar{b}$ to $\text{span}\{\bar{w}_1, \bar{w}_2\}$

Recall: $\mathbf{w}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{w}_2 = \begin{bmatrix} 2 \\ 4 \\ 2 \end{bmatrix}$ and let $\mathbf{b} = \begin{bmatrix} 4 \\ 0 \\ 8 \end{bmatrix}$.

(c) Let A be the matrix $A = [\mathbf{w}_1\ \mathbf{w}_2]$. Decide whether $A\mathbf{x} = \mathbf{b}$ is consistent and explain your answer.

$$[A\bar{b}] = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 4 & 0 \\ 1 & 2 & 8 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 4 \\ 0 & 6 & 4 \\ 0 & 0 & 4 \end{bmatrix}$$

inconsistent as $0=4$ is one of the equations (pivot in last col of augmented matrix)

(d) Find all least-squares solutions to $A\mathbf{x} = \mathbf{b}$.

$$A^T A \hat{x} = A^T \bar{b}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 4 & 2 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix}\hat{x} = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 4 & 2 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 8 \end{bmatrix}, \quad \begin{bmatrix} 3 & 0 \\ 0 & 24 \end{bmatrix}\bar{x} = \begin{bmatrix} 12 \\ 24 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 0 & 12 \\ 0 & 24 & 24 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 1 \end{bmatrix} \quad \hat{x} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

alternate method! Since the columns of A are orthogonal and nonzero you can use the formula

$$\bar{x} = \begin{bmatrix} \frac{\bar{a}_1 \cdot \bar{b}}{a_1 \cdot \bar{a}_1} \\ \frac{\bar{a}_2 \cdot \bar{b}}{a_2 a_2} \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

Note that the normal equations can always be used to find least squares solutions

10. (6 points) Let $\mathbf{x}_1 = \begin{bmatrix}1\\0\\1\\0\end{bmatrix}$, $\mathbf{x}_2 = \begin{bmatrix}2\\0\\0\\2\end{bmatrix}$, $\mathbf{x}_3 = \begin{bmatrix}0\\0\\8\\8\end{bmatrix}$. Use the Gram-Schmidt process to find an orthogonal basis of $W = \text{Span}\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$. You may assume that $\{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3\}$ is a basis of W.

$$\bar{V}_1 = \bar{X}_1 = \begin{bmatrix}1\\0\\1\\0\end{bmatrix}$$

$$\bar{V}_2 = \bar{X}_2 - \text{proj}_{\text{span}\{\bar{V}_1\}} \bar{X}_2 = \begin{bmatrix}2\\0\\0\\2\end{bmatrix} - \underbrace{\frac{\begin{bmatrix}2\\0\\0\\2\end{bmatrix}\cdot\begin{bmatrix}1\\0\\1\\0\end{bmatrix}}{2}}_{=1}\begin{bmatrix}1\\0\\1\\0\end{bmatrix} = \begin{bmatrix}1\\0\\-1\\2\end{bmatrix}$$

$$\bar{V}_3 = \bar{X}_3 - \text{proj}_{\text{span}\{\bar{V}_1, \bar{V}_2\}} \bar{X}_3 = \begin{bmatrix}0\\0\\8\\8\end{bmatrix} - \left(\underbrace{\frac{\begin{bmatrix}0\\0\\8\\8\end{bmatrix}\cdot\begin{bmatrix}1\\0\\1\\0\end{bmatrix}}{2}}_{=4}\begin{bmatrix}1\\0\\1\\0\end{bmatrix} + \underbrace{\frac{\begin{bmatrix}0\\0\\8\\8\end{bmatrix}\cdot\begin{bmatrix}1\\0\\-1\\2\end{bmatrix}}{6}}_{=4/3}\begin{bmatrix}1\\0\\-1\\2\end{bmatrix}\right)$$

$$= \begin{bmatrix}0\\0\\8\\8\end{bmatrix} - \left(\begin{bmatrix}4\\0\\4\\0\end{bmatrix} + \begin{bmatrix}4/3\\0\\-4/3\\8/3\end{bmatrix}\right)$$

$$= \begin{bmatrix}-16/3\\0\\16/3\\16/3\end{bmatrix}$$

(Note that any nonzero multiple of this vector would still be orthogonal to $\bar{v}_1, \bar{v}_2$, eg $\begin{bmatrix}-1\\0\\1\\1\end{bmatrix}$)

11. (8 points) Let $\mathbf{w}_1$ and $\mathbf{w}_2$ be vectors in $\mathbb{R}^3$. Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(\mathbf{v}) = \begin{bmatrix} \mathbf{v} \cdot \mathbf{w}_1 \\ \mathbf{v} \cdot \mathbf{w}_2 \end{bmatrix}$.

Prove that T is linear.

Let $\bar{u}$ and $\bar{v}$ be vectors in $\mathbb{R}^3$ and $c \in \mathbb{R}$

$$T(\bar{u}+\bar{v}) = \begin{bmatrix} (\bar{u}+\bar{v})\cdot \bar{w}_1 \\ (\bar{u}+\bar{v})\cdot \bar{w}_2 \end{bmatrix} = \begin{bmatrix} \bar{u}\cdot\bar{w}_1 + \bar{v}\cdot\bar{w}_1 \\ \bar{u}\cdot\bar{w}_2 + \bar{v}\cdot\bar{w}_2 \end{bmatrix}$$

by def T by rules of dot product.

$$= \begin{bmatrix} \bar{u}\cdot\bar{w}_1 \\ \bar{u}\cdot\bar{w}_2 \end{bmatrix} + \begin{bmatrix} \bar{v}\cdot\bar{w}_1 \\ \bar{v}\cdot\bar{w}_2 \end{bmatrix} = T(\bar{u}) + T(\bar{v})$$

$$T(c\bar{u}) = \begin{bmatrix} (c\bar{u})\cdot\bar{w}_1 \\ (c\bar{u})\cdot\bar{w}_2 \end{bmatrix} = \begin{bmatrix} c(\bar{u}\cdot\bar{w}_1) \\ c(\bar{u}\cdot\bar{w}_2) \end{bmatrix} = c\begin{bmatrix} \bar{u}\cdot\bar{w}_1 \\ \bar{u}\cdot\bar{w}_2 \end{bmatrix}$$

$$= c\,T(\bar{u})$$

$\therefore T$ is linear

12. (8 points) Let V and W be vector spaces and let $T : V \to W$ be a linear transformation *that is one-to-one*. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a linearly independent set of a vectors in V. Prove that the set $\{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ is linearly independent in W.

We check the definition of independence

Let c_1, c_2, c_3 be weights

assume $c_1 T(\bar{v}_1) + c_2 T(\bar{v}_2) + c_3 T(\bar{v}_3) = \bar{0}$

by linearity $T(c_1\bar{v}_1 + c_2\bar{v}_2 + c_3\bar{v}_3) = \bar{0}$ (use hypothesis T is linear)

As T is one-to-one, the only solution to $T(\bar{v}) = \bar{0}$ is $\bar{v} = \bar{0}$ (use hypothesis T is one-to-one)

$\therefore$ $c_1\bar{v}_1 + c_2\bar{v}_2 + c_3\bar{v}_3 = \bar{0}$

As $\{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ is independent (use hypothesis $\{\bar{v}_1, \bar{v}_2, \bar{v}_3\}$ is indep)

$c_1 = c_2 = c_3 = 0$

$\therefore$ $\{T(\bar{v}_1), T(\bar{v}_2), T(\bar{v}_3)\}$ is indep.

13. (8 points) Let W be a subspace of $\mathbb{R}^n$.
Its orthogonal complement is $W^\perp = \{\mathbf{x} \in \mathbb{R}^n \mid \mathbf{x} \cdot \mathbf{w} = 0 \text{ for all } \mathbf{w} \in W\}$.
Use the definition of subspace to prove that $W^\perp$ is a subspace of $\mathbb{R}^n$.

(1) $\bar{0} = \bar{0}_{\mathbb{R}^n} \in W^\perp$ as $\bar{0} \cdot \bar{w} = 0 \ \forall \bar{w} \in W$

(2) Let $\bar{u} \in W^\perp$ an $\bar{v} \in W^\perp$

$\boxed{\begin{array}{l} \text{so } \bar{u} \cdot \bar{w} = 0 \ \forall \bar{w} \in W \\ \text{so } \bar{v} \cdot \bar{w} = 0 \ \forall \bar{w} \in W \end{array}}$

write explicitly what your assumptions $\bar{u} \in W^\perp$, $\bar{v} \in W^\perp$ mean

So if $\bar{w} \in W$, $(\bar{u} + \bar{v}) \cdot \bar{w} = \bar{u} \cdot \bar{w} + \bar{v} \cdot \bar{w} = 0 + 0 = 0$
as $\bar{u} \in W^\perp$ $\bar{v} \in W^\perp$
$\therefore \bar{u} + \bar{v} \in W^\perp$

3 Let $\bar{u} \in W^\perp$ $c \in \mathbb{R}$

so if $\bar{w} \in W$, $(c\bar{u}) \cdot \bar{w} = c(\bar{u} \cdot \bar{w}) = 0$
as $\bar{u} \in W^\perp$

$\therefore c\bar{u} \in W^\perp$

$\therefore W^\perp$ is a subspace of $\mathbb{R}^n$

End of Test. Please fill in the information on the next page.
Have a great summer!

Math 70 **Final Exam** **May 9, 2016**

Name: ____

Circle the name of your instructor

Jessica Dyer

Mary Glaser Glaser's class: 4-digit secret code which I will use to post grades: _______

Hao Liang

Todd Quinto

I pledge that I have neither given nor received assistance on this exam.

Signature ____