Math 70 **TUFTS UNIVERSITY** **April 4, 2016**

Linear Algebra **Department of Mathematics**

Exam II

Instructions: No notes or books are allowed. All calculators, cell phones, or other electronic devices **must** be turned off and put away during the exam. Unless otherwise stated, you **must show all work** to receive full credit. *You are required to sign your exam. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

Problem	Point Value	Points
1	6	
2	4	
3	14	
4	8	
5	6	
6	10	
7	10	
8	10	
9	10	
10	8	
11	6	
12	8	
	100	

1. (6 pts) Let $A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$. Is A invertible? If so, find A^{-1}.

2. (4 pts) Let V be a vector space and let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n\}$ be an indexed set of vectors in V. Complete the following: B is a basis for V if

3. (14 pts) Let V and W be vector spaces.

   (a) Let $T : V \to W$ be a transformation. Give the conditions from the definition for T to be a *linear* transformation.

   (b) Define the transformation $T : \mathbb{P}_2 \to \mathbb{R}_2$ by $T(p(t)) = \begin{bmatrix} p(2) \\ p(0) \end{bmatrix}$.

      (i) Use your definition from part (a) to prove T is a linear transformation.

The map $T : \mathbb{P}_2 \to \mathbb{R}_2$ is given by $T(p(t)) = \begin{bmatrix} p(2) \\ p(0) \end{bmatrix}$.

(ii) Find a basis for the kernel of T.

4. (8 pts) Consider the polynomials $\mathbf{p}_1(t) = 1 + t, \quad \mathbf{p}_2(t) = t + t^2, \quad \mathbf{p}_3(t) = 3 + 3t + t^2.$ Determine whether or not $S = \{\mathbf{p}_1(t), \mathbf{p}_2(t), \mathbf{p}_3(t)\}$ is linearly independent in $\mathbb{P}_2$. Explain your procedure.

5. (6 pts) Let V be the vector space of all continuous, real-valued functions. Let H be the subspace of V with a basis $\mathcal{B} = \{e^t, \cos(t), t^2, 1\}$.

   (a) The dimension of H is ______.

   (b) Find $[4 + 2\cos(t) + e^t + 3t^2]_{\mathcal{B}}$.

6. (10 pts) Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ where $\mathbf{b}_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

   (a) Find the vector $\mathbf{x}$ whose coordinate vector relative to $\mathcal{B}$ is $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$.

   (b) Suppose $\mathbf{x} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$. Find the coordinates of $\mathbf{x}$ relative to $\mathcal{B}$ - i.e. find $[\mathbf{x}]_{\mathcal{B}}$.

7. (10 pts) Let $A = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 3 & 4 & 5 \\ 5 & 4 & 0 & 3 \\ 2 & 0 & 0 & 1 \end{pmatrix}$. Compute $\det(A)$ and use the result to decide whether or not A is invertible.

8. (10 pts) Let $A = \begin{bmatrix} 1 & 0 & 0 & 0 & 3 \\ 0 & 0 & 1 & 2 & -4 \\ 1 & 0 & 1 & 2 & -1 \end{bmatrix}$.

Find a basis for each of the following spaces (show your work) :

(a) $\text{Nul}(A)$

(b) $\text{Col}(A)$

9. (10 pts)

(a) Let H be a subset of a vector space V. Give the conditions from the definition for H to be a subspace.

(b) Use the definition from part (a) to show that $H = \{a + bt^2 \in \mathbb{P}_2 \mid a + b = 0\}$ is a subspace of $\mathbb{P}_2$.

10. (8 points) Suppose a nonhomogeneous system of 10 linear equations in 12 unknowns is *inconsistent*. Is it possible to find 3 solutions of the associated homogeneous system that are linearly independent? Explain.

11. (6 pts) Suppose that the columns of a $n \times n$ matrix B are linearly dependent. Show that the columns of AB are also linearly dependent for any $n \times n$ matrix A.

    (Hint: the columns of a $n \times n$ matrix C are linearly independent if and only if $\det(C) \neq 0$.)

12. (8 pts) Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ be a basis for a vector space V. Prove that $\mathcal{B}' = \{\mathbf{b}_1 + \mathbf{b}_2, \mathbf{b}_1 - \mathbf{b}_2\}$ is also a basis for V. Explain your procedure and show all work.

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Name ______________________________________

Please circle the name of your instructor

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I pledge that I have neither given nor received assistance on this exam.

Signature ___________________________________