Math 70	**TUFTS UNIVERSITY**	**November 16, 2015**
Linear Algebra	**Department of Mathematics**	**Sections 1 and 2**
	Exam II	

Instructions: No notes or books are allowed. All calculators, cell phones, or other electronic devices **must** be turned off and put away during the exam. Unless otherwise stated, you **must show all work** to receive full credit. **Please reference $\vec{0}$ with the vector space to which it belongs, e.g. the zero vector in V should be subscripted O_V.** *You are required to sign your exam.With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

Problem	Point Value	Points
1	10	
2	12	
3	13	
4	9	
5	8	
6	10	
7	10	
8	10	
9	10	
10	8	
	100	

1. (10 points)

$$\text{Let } A = \begin{pmatrix} 1 & 1 & 4 & 1 & 2 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 \\ 1 & -1 & 0 & 0 & 2 \\ 2 & 1 & 6 & 0 & 1 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 2 & 0 & -1 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Then A and B are row equivalent. (You do not need to show this.)

(a) Find a basis $\mathcal{B}$ for Col A.

(b) Show that Col $A \neq$ Col B by finding a vector $\mathbf{v}$ that is in Col A but not Col B. Briefly justify.

(c) Find the dimension of Nul A.

2. (12 pts) For this whole problem let V be a vector space with basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$.

   (a) V is isomorphic to $\mathbb{R}^n$ where $n =$ _____.

   (b) Using the n you found above, prove that the mapping $T : V \to \mathbb{R}^n$ given by $T(\mathbf{v}) = [\mathbf{v}]_{\mathcal{B}}$ is linear.

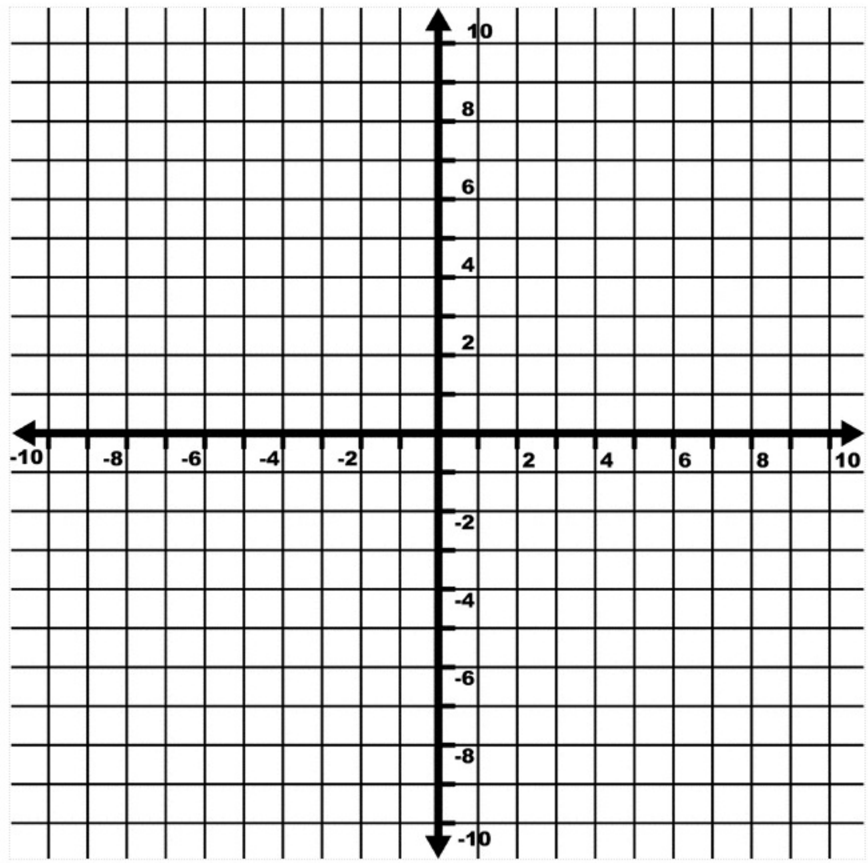
3. (13 pts) Let $T : V \to W$ be linear where V and W are vector spaces.

   (a) Define the set ker T, the kernel of T.

   (b) Ker T is a subset of the vector space _____.

   (c) Prove that ker T is a subspace of the vector space you named above.

4. (9 pts) Let A be a 2×2 matrix. Let T be the map $x \mapsto Ax$, i.e. the map $T(x) = Ax$. Plot and *label* the following vectors in the given plane.



(a) $T(x)$ where $x = (4, 2)$ is an eigenvector of A with eigenvalue $\lambda = -2$.

(b) An eigenvector $u \neq x$ in the same eigenspace as x above.

(c) $T(y)$ where $y = (-2, 3)$ is an eigenvector of A with eigenvalue $\lambda = 3$.

5. (8 pts) Let $A = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 3 & 2 \\ -2 & 0 & 1 \end{pmatrix}$.

Find a basis for the eigenspace of A corresponding to the eigenvalue $\lambda = 3$.

This is equivalent to finding a basis for Nul (__________). (fill in the blank.)

6. (10 pts) Let $S = \{p_1 = 1 + t^2,\ p_2 = t + t^2,\ p_3 = 2 - 3t - t^2,\ p_4 = 4 + t + 5t^2\}$ be a set of polynomials in $\mathbb{P}_2$.

   (a) The dimension of $\mathbb{P}_2$ is _____ so $\mathbb{P}_2$ is isomorphic to R—. (fill in the blanks)

   (b) Do the polynomials in S span $\mathbb{P}_2$? Justify completely by explaining your method.

7. (10 pts) Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ where $\mathbf{b}_1 = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

(a) Find $P_{\mathcal{B}}$, the change-of-coordinates matrix from $\mathcal{B}$ to the standard basis in $\mathbb{R}^2$.

(b) Suppose $\mathbf{x} = \begin{bmatrix} 8 \\ 2 \end{bmatrix}$. Find the coordinates of $\mathbf{x}$ relative to $\mathcal{B}$, i.e. find $[\mathbf{x}]_{\mathcal{B}}$.

8. (10 pts) Let $A = \begin{pmatrix} 1 & 2 & 0 & 2 \\ 0 & 0 & 0 & -2 \\ 1 & 0 & 1 & 0 \\ 1 & 3 & -1 & 0 \end{pmatrix}$.

(a) Find $\det A$.

(b) Find $\det(2A^3)$.

9. (10 pts) Let $T : \mathbb{R}^4 \to \mathbb{M}_{2\times 3}$ be defined by

$$T\left(\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\right) = \begin{bmatrix} 3a & a+c & 0 \\ 0 & 0 & 2c \end{bmatrix}.$$

Then T is linear. (You do not need to show this.)

(a) Find a basis for the range of T. Show work but no justification of basis properties is needed.

(b) Find a basis for ker T. Show work but no justification of basis properties is needed.

(c) Is T one-to-one? Briefly justify your answer.

10. (8 pts) A linear transformation is called *onto* if for every vector $w \in W$ there is at least one vector $v \in V$ with $T(v) = w$. Suppose that the linear transformation $T : V \to W$ is onto. Let $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a spanning set for V. Show that $S' = \{T(v_1), T(v_2), T(v_3)\}$ is a spanning set for W. The first line of your proof should be: "Let w be any vector in W."

Math 70 Exam II November 16, 2015 Sections 1 and 2

Name ____

Please circle your section

Section 1 Hao Liang

Section 2 Mary Glaser

I pledge that I have neither given nor received assistance on this exam.

Signature ____________________________________