Math 70	**TUFTS UNIVERSITY**	**March 30, 2015**
Linear Algebra	**Department of Mathematics**	**Sections 1 and 2**
	Exam II	

Instructions: No notes or books are allowed. All calculators, cell phones, or other electronic devices **must** be turned off and put away during the exam. Unless otherwise stated, you **must show all work** to receive full credit. *You are required to sign your exam. With your signature you are pledging that you have neither given nor received assistance on the exam. Students found violating this pledge will receive an F in the course.*

Problem	Point Value	Points
1	10	
2	8	
3	16	
4	12	
5	8	
6	8	
7	10	
8	10	
9	10	
10	8	
	100	

1. (10 pts) **True/false questions.** Decide whether each of the statements below is true or false. Indicate your answer by shading the appropriate box. No explanations needed. No partial credit.

   (a) For square matrices A and B, provided AB and BA are defined, $|AB| - |BA| = 0$. T F

   (b) For a square invertible matrix A, $\det(A^{-1}) = \frac{1}{\det A}$. T F

   (c) For any square matrix A, if $|A| \neq 0$ then A is row equivalent to I. T F

   (d) If A is an invertible matrix, then for any square matrix B for which AB is defined, AB is also invertible. T F

   (e) If A and B are square matrices and $AB = B$, then A = I. T F

2. (8 pts) It is a fact that $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 1 & -3 \\ 0 & 1 & -1 \end{bmatrix}$ are inverses of each other.

   Use this fact to solve the system of equations:

$$\begin{array}{rcrcrcl} & - & y & + & 2z & = & 2 \\ x & + & y & - & 3z & = & 5 \\ & & y & - & z & = & 2 \end{array}$$

3. (16 pts) Let V and W be vector spaces.

   (a) Let $T : V \to W$ be a transformation. Give the conditions from the definition for T to be a *linear* transformation.

   (b) Define the transformation $T : \mathbb{M}_{2\times 2} \to \mathbb{P}_2$ by $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + dt^2$.

      (i) Use your definition from part (a) to prove T is a linear transformation.

$$T : \mathbb{M}_{2\times 2} \to \mathbb{P}_2 \text{ by } T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + dt^2.$$

(ii) Find a basis for the kernel of T.

(iii) Find a basis for the range of T.

4. (12 pts) Consider the polynomials $\mathbf{p}_1(t) = 1 + t, \mathbf{p}_2(t) = 2 + t^2, \mathbf{p}_3(t) = 5 + 3t + t^2$.

(a) Is the set $S = \{\mathbf{p}_1(t), \mathbf{p}_2(t), \mathbf{p}_3(t)\}$ linearly independent in $\mathbb{P}_2$? Explain.

(b) Find a basis for the subspace of $\mathbb{P}_2$ spanned by S.

5. (8 pts) Let V be the vector space of all continuous, real-valued functions. Let H be the subspace of V with basis $\mathcal{B} = \{e^t, \cos(t), \sin(t), 1\}$.

   (a) H is isomorphic to $\mathbb{R}^n$ where $n =$______ .

   (b) Find $[3 + 2\cos(t) + 5e^t]_{\mathcal{B}}$.

6. (8 pts) Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ where $\mathbf{b}_1 = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$ and $\mathbf{b}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

   (a) Find the vector $\mathbf{x}$ whose coordinate vector relative to $\mathcal{B}$ is given by: $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$

   (b) Suppose $\mathbf{x} = \begin{bmatrix} 4 \\ 0 \end{bmatrix}$. Find the coordinates of $\mathbf{x}$ relative to $\mathcal{B}$ - i.e. find $[\mathbf{x}]_{\mathcal{B}}$.

7. (10 pts) For $A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & 1 \\ 1 & 3 & 2 \end{bmatrix}$, find a basis for each of the following spaces:

(a) Nul(A)

(b) Col(A)

8. (10 pts) Compute $\begin{vmatrix} 1 & 0 & 0 & 2 \\ 0 & 3 & 4 & 5 \\ 5 & 4 & 0 & 3 \\ 2 & 0 & 0 & 1 \end{vmatrix}$ and use result to decide whether or not the matrix is invertible.

9. (10 pts)

(a) Let H be a subset of a vector space V. Give the conditions from the definition for H to be a subspace.

(b) Use the definition from part (a) to show that $\left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \;\middle|\; a+b=0 \right\}$ is a subspace of $\mathbb{M}_{2\times 2}$.

10. (8 points) Let $T : V \longrightarrow W$ be an one-to-one linear transformation.
Suppose that $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ is a linearly independent subset of V.
Prove that $S' = \{T(\mathbf{v}_1), T(\mathbf{v}_2), T(\mathbf{v}_3)\}$ is a linearly independent subset of W.

Math 70 Exam II March 30, 2015 Sections 1 and 2

Name ____

Please circle your section

Section 1 Hao Liang

Section 2 Mary Glaser

I pledge that I have neither given nor received assistance on this exam.

Signature ______________________________________