

MATH 61-02: WORKSHEET 9 (§6.1-6.4)

(W1) Show that \mathbb{Z}^3 is countable.

(W2) Suppose that you're walking on a road and every mile you go you come to another fork in the road. Suppose the roads go on forever. Consider the set P of all paths (that is, all choices of L or R at every fork; so one path is LLLLLL... and another path is LRLRRLRRRLRRRR....). Prove that $\aleph_0 < |P| \leq c$, where $\aleph_0 = |\mathbb{Z}|$ and $c = |\mathbb{R}|$. (Bonus: prove $|P| = c$.)

(W3) Prove that $|(0, 1)| = |[0, 1]|$, or in other words, there is a bijection between the open interval and the closed interval. (Hint: by Schroeder-Bernstein, it suffices to find injections both ways.)

(W4) **Bonus question (extra credit):** A monster is moving in a straight line in \mathbb{R}^2 at a constant speed so that every minute, they arrive at a lattice point. Suppose every minute you can place one bomb at a lattice point that will kill the monster if it is there. You don't know where the monster started or where it is. Can you strategically place bombs in such a way that is guaranteed to eventually kill the monster?