MATH 61-02: WORKSHEET 5 (§4.1-§4.3)

(W1) Everyone in your 42-person Discrete Math class goes to either the Republican or the Democratic presidential primary. In the Republican presidential primary, Kasich, Trump, Rubio, and Cruz are running. In the Democratic presidential primary, Clinton, Sanders, and O'Malley are running.

(a) Suppose that everyone chooses one of the party primaries, and then chooses their favourite candidate running in that party. How many ways can the choices go?

(b) Suppose that everyone is asked to rank all seven candidates in order of preference. Moon asks Kris, "How many ways can the choices go?", to which he replies, "42(7!)". Is he right, and why?

(c) Suppose that the Democratic ballot asks voters to rank the Dem candidates in order of preference, but the Republican ballot requires a choice of top two Republicans, unranked. How many ways can the class's choices go?

(d) Suppose that everyone at both primaries is asked to rank the candidates running in that party in order of preference, but also asked to pick their least favourite candidate running from the other party. How many ways can the choices go now?

(W2) *Totó* is a form of gambling in Hungary. On one ticket, you can try to bet on the outcomes of 13 soccer matches in order - for each match, you bet whether the home team will win, the away team will win, or whether the match will end in a draw. A sample ticket looks like this:

Match	1	2	3	4	5	6	7	8	9	10	11	12	13
Bet	H	D	A	A	A	D	A	H	A	D	D	D	H

(a) How many different tickets can you buy?

(b) How many different tickets can you buy where no two consecutive bets are the same?

(c) How many different tickets can you buy where you bet H exactly three times?

(d) How many different tickets can you buy where you bet that the home team will win exactly three times, the away team will win exactly six times, and the teams will draw exactly four times?

(e) How many different tickets can you buy where you bet that the home team will win either seven or eight times?

(f) Consider the string of letters formed by your bet (for instance, "HDAAADAHADDDH" on the ticket above). How many different tickets can you buy where your bet string is a palindrome (i.e., reading the same forwards or backwards)?

(g) Combine parts (e) and (f): How many different tickets can you buy where the bet string is a palindrome with either 7 or 8 H's?

(h) Suppose you're given a tip-off that the home team will not win any prime-numbered game and the away team will win any game whose number is a multiple of 3. How many different tickets are consistent with this information?

(i) **Bonus question (Email your solution to** kristofer.siy@tufts.edu **for extra credit):** How many tickets do you need to buy to ensure that one of your tickets will have five correct predictions? Prove that the answer you give is the minimum number of tickets for which this is true. (N.B. I'll accept solutions to this problem until March 7.)

(W3) (a) Let's say that a "diagonal" of a polygon is a straight line from vertex to vertex that is not equal to an edge of the polygon. For instance, a triangle has no diagonals and a square has two. How many diagonals does a regular n-gon have (where $n \geq 3$)? Prove your answer using:

(i) counting. (Hint: each vertex is connected to how many others? And how many times does this count each edge?)

(ii) induction.

(b) Suppose that $m, n \geq 2$. Consider $S = \{(p, q) \mid 1 \leq p \leq m, 1 \leq q \leq n\}$, an $m \times n$ array of lattice points. Moon asks Kris, "How many ways are there to choose four points from this array that are the corners of a rectangle with sides parallel to the $x-$ and $y-$axes?" Kris answers "I count $\binom{m}{2} \cdot \binom{n}{2}$." Is he right, and why?