MATH 61-02: WORKSHEET 3 (§2.1)

(W1) Let $S$ be a subset of $[9]$ such that $|S| = 6$. Show that $\exists a, b \in S$ s.t. $a + b = 10$.

(W2) Consider the sequence 7, 87, 887, 8887, \ldots Show that no number in this sequence is the sum of three squares.
(W3) We are going to play a game where I think of a number between 1 and $N$; each time you guess a number, I will tell you if it is high or if it is low. Suppose $N$ is at least 1000. Come up with a strategy that will guarantee you find the number in fewer than $N/10$ steps.

(W4) (a) Find all solutions to the equation $x^2 = 3y + 2$ for $x, y \in \mathbb{N}$. (With proof, of course.)

(b) (Bonus—Extra Credit) Prove that $x^2 + y^2 = 3z^2$ has no solutions $x, y, z \in \mathbb{N}$. 