

**MATH 61-02: WORKSHEET 3 (§2.1)**

(W1) Let  $S$  be a subset of  $[9]$  such that  $|S| = 6$ . Show that  $\exists a, b \in S$  s.t.  $a + b = 10$ .

(W2) Consider the sequence  $7, 87, 887, 8887, \dots$ . Show that no number in this sequence is the sum of three squares.

(W3) We are going to play a game where I think of a number between 1 and  $N$ ; each time you guess a number, I will tell you if it is high or if it is low. Suppose  $N$  is at least 1000. Come up with a strategy that will guarantee you find the number in fewer than  $N/10$  steps.

(W4) (a) Find all solutions to the equation  $x^2 = 3y + 2$  for  $x, y \in \mathbb{N}$ . (With proof, of course.)

(b) (Bonus—Extra Credit) Prove that  $x^2 + y^2 = 3z^2$  has no solutions  $x, y, z \in \mathbb{N}$ .