MATH 61-02: WORKSHEET 11 (§7.1)

(W1) (a) For a vertex v in a graph $G = (V, E)$, let $\deg(v)$ be the *degree* of v, which is the number of times v appears as the endpoint of an edge. (So loops count double.) What is the smallest possible graph with at least one loop where every vertex has odd degree?

(b) Let K_n be the complete graph on n vertices (the simple graph where each vertex is connected by one edge to each other vertex). Sketch K_4, K_5, and K_6. For general n, what is $|E(K_n)|$ and what is the degree of each vertex?

(c) Prove that $\sum_{v \in V} \deg(v) = 2|E|$ for any graph.

(d) Is it possible for a graph to have 11 vertices, all of which have degree 3?

(e) Is it possible for a graph to have 19 vertices, each of which have degree 1, 5, or 9? (Hint: consider the degree sum mod 4.)

(W2) Suppose G is a simple graph (no loops) with $|V(G)| = n$. Show that if the degree of every vertex in G is at least $\frac{n-1}{2}$, then G is connected. (First convince yourself this is true for $n = 2, 3, 4$.)