

Name: \_\_\_\_\_

Date: \_\_\_\_\_

Quiz name: Discrete Math Apr 13 (more cardinality)

1. Which of these techniques can be used to show that  $|\mathbb{R}| = |\mathbb{R}^4|$  ?

- (A) Diagonalization / the barber of Seville paradox
- (B) Snake enumeration / space-filling curves
- (C) Power sets / towers of infinity
- (D) Disjoint unions / sums of cardinals

2. Suppose  $f$  maps  $X$  to  $Y$  and  $g$  maps  $Y$  to  $Z$ . If the composition of these two functions is a surjection, what must be true of the individual functions?

- (A)  $f$  must be surjective and  $g$  must be injective.
- (B) At least one must be a bijection.
- (C) They must both be surjections.
- (D) Only one of  $f$  and  $g$  needs to be surjective.

Only  $g$  needs to be surjective! Example:  
 $X=Y$  is the integers, and  $Z=\{1\}$ ; let  $f(x)=x^2$   
and let  $g(y)=1$  for all  $y$ . Then the first one is  
not surjective, but the composition is.

3. Consider the Hilbert curve  $H:[0,1] \rightarrow [0,1] \times [0,1]$ . Let  $S$  be the set of rational points in  $[0,1]$ . What is the cardinality of the image  $H(S)$ ?

- (A) A large finite number.
- (B) Aleph-nought
- (C)  $c$
- (D)  $c^2$ , which is bigger than  $c$  by Cantor's theorem
- (E) Undecidable; it depends on the Continuum Hypothesis.