1) a) Integrate twice to get $x = \frac{t^3}{2} + \frac{t^2}{2} + At + B$.

b) $x' = \frac{3}{2}t^2 + t + A \qquad x'(0) = A = 3$

once you have $A = 3 \qquad x(0) = B = 2$

Your answer should be $x = \frac{t^3}{2} + \frac{t^2}{2} + 3t + 2$

2) $(D-1)x = 3$ Homogeneous solutions: $h = Ke^t$ K arbitrary

Try a constant particular solution $p(t) = c \qquad Dp = 0$

So $-c = 3 \qquad c = -3 = p(t)$

Your answer should be $x = -3 + Ke^t$ K arbitrary

3
$$W = \begin{vmatrix} e^t & \sin t & \cos t \\ e^t & \cos t & -\sin t \\ e^t & -\sin t & -\cos t \end{vmatrix} = e^t \begin{vmatrix} 1 & \sin t & \cos t \\ 1 & \cos t & -\sin t \\ 1 & -\sin t & -\cos t \end{vmatrix}$$

subtract 1st row from 2nd and 3rd.

$$= e^t \begin{vmatrix} 1 & \sin t & \cos t \\ 0 & \cos t - \sin t & -\sin t - \cos t \\ 0 & -2\sin t & -2\cos t \end{vmatrix} = e^t\left[2\cos t(\sin t - \cos t) \cancel{+} 2\sin t(\sin t + \cos t)\right]$$

$$= e^t\left[2\cos t \sin t - 2\cos^2 t \cancel{+} 2\sin^2 t - 2\cos t \sin t\right] = -2e^t \neq 0$$

∴ functions are indep.

4) Clear up first column

$$\left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 1 \\ 0 & -3 & -6 & 1 & 0 \\ 0 & -3 & -6 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 3 & 0 & 1 \\ 0 & 1 & 2 & -1/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

two identical rows
divide by -3

to reduce need to clear up second column

$$\left(\begin{array}{cccc|c} 1 & 0 & -1 & 2/3 & 1 \\ 0 & 1 & 2 & -1/3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right)$$ reduced.

Solution $\vec{x} = z\begin{pmatrix} 1 \\ -2 \\ 1 \\ 0 \end{pmatrix} + w\begin{pmatrix} 2/3 \\ -1/3 \\ 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

5) $x = Ae^{-2t} + Bte^{-2t} + Ct^2e^{-2t} + Ee^{t}$

6) $5r^2 + 2r + 1 = 0$ gives $r = \frac{-2 \pm \sqrt{4-20}}{10} = -\frac{1}{5} \pm \frac{2}{5}i$

So solution is generated by $e^{-t/5}\cos\frac{2}{5}t$ and $e^{-t/5}\sin\frac{2}{5}t$

7) Annihilator $(D+2)^2(D^2+1)$

New Eq $(D+2)^3(D^2+1)^2 x = 0$

Old homogeneous solution: $h = Ae^{-2t} + B\cos t + C\sin t$

New homogeneous solution $x = Ae^{-2t} + A_1te^{-2t} + A_2t^2e^{-2t} +$

$B\cos t + B_1 t\cos t + C\sin t + C_1 t\sin t.$

Simplified guess $A_1te^{-2t} + A_2t^2e^{-2t} + B_1t\cos t + C_1 t\sin t$

8) a) $x = t^\alpha$ plugged in gives

$\alpha(\alpha-1)t^{\alpha-1} - \alpha t^{\alpha-1} = (\alpha^2 - 2\alpha)t^{\alpha-1}$ so $\alpha^2 - 2\alpha = 0$

This gives two solutions; $\alpha = 0$, $\alpha = 2$.

General homogeneous solution $h = C_1 + C_2t^2$

b) Use V. P. Wronskian is $\begin{vmatrix} 1 & t^2 \\ 0 & 2t \end{vmatrix} = 2t$; standard eq is $(D^2 - \frac{1}{t}D)x = t$

$$c_1' = \frac{\begin{vmatrix} 0 & t^2 \\ t & 2t \end{vmatrix}}{2t} = -\frac{t^2}{2} \qquad c_1 = -\frac{t^3}{6}$$

$$c_2' = \frac{\begin{vmatrix} 1 & 0 \\ 0 & t \end{vmatrix}}{2t} = \frac{1}{2} \qquad c_2 = \frac{t}{2}$$

Part Solution: $-\frac{t^3}{6}\cdot 1 + \frac{t}{2}\cdot t^2 = \frac{1}{3}t^3$

General Solution $x = \frac{1}{3}t^3 + C_1 + C_2t^2$ C_1, C_2 arbitrary constants.