Midterm Exam 3

Instructions. No calculators, notes, or books are allowed. Please make sure all electronic devices are turned off and out of sight. Show all work and cross out work you do not want graded! *Remember to sign your blue book.* With your signature you are pledging that you have neither given nor received assistance on this exam.

Good luck!

1. (10 points): Check following sets of vectors for independence (NO PARTIAL CREDIT):

(a) $\begin{pmatrix}1\\7\\2\end{pmatrix}, \begin{pmatrix}2\\4\\-1\end{pmatrix}, \begin{pmatrix}1\\0\\-1\end{pmatrix}, \begin{pmatrix}2\\0\\-1\end{pmatrix}$

(b) $\begin{pmatrix}4\\3\\2\\1\end{pmatrix}, \begin{pmatrix}-2\\2\\4\\8\end{pmatrix}, \begin{pmatrix}6\\8\\8\\10\end{pmatrix}$

2. (15 points): (a) Find the eigenvalues of the matrix $A = \begin{pmatrix}1 & 0 & 0\\-1 & 2 & 0\\0 & 1 & 3\end{pmatrix}$

(b) Find eigenvectors for each eigenvalue of the matrix A.

3. (10 points): The matrix $A = \begin{pmatrix}1 & 1 & 0\\1 & 0 & 1\\1 & 1 & 0\end{pmatrix}$ has eigenvectors $v = \begin{pmatrix}1\\1\\1\end{pmatrix}$ and $w = \begin{pmatrix}1\\-2\\1\end{pmatrix}$.

Find the corresponding eigenvalues.

4. (10 points): Use the row reduction method to solve the following systems:

$$\begin{aligned} x_1 + 2x_2 + x_3 - x_4 - x_5 &= 2 \\ 2x_1 + 2x_2 + 2x_3 - 3x_4 - 2x_5 &= 1 \\ -x_1 \qquad\quad - x_3 + 2x_4 + x_5 &= 1. \end{aligned}$$

No credit for any other method.

5. (20 points): Solve the system of linear differential equations $D\vec{x} = A\vec{x}$, where

$$A = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & -2 & 0 \end{pmatrix}$$

6. (15 points): The matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

has a triple eigenvalue $\lambda = 1$ (You do not have to verify this). Find the generalized eigenvectors of A.

7. (20 points): Solve the following initial value problem:

$$D\vec{x} = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix} \vec{x} \qquad\qquad \vec{x}(0) = \begin{pmatrix} 10 \\ 20 \\ 10 \end{pmatrix}$$