Mathematics 51 **Tufts UNIVERSITY** **Differential Equations**
Examination II **April 8, 2013, 12–1:20**

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. **You must show all your work** in the blue book in order to receive full credit. *Please box your answers and cross out any work you do not want graded.* Make sure to sign your blue book. With your signature you are pledging that you have neither given nor received assistance on the exam. *Good luck!*

1. (5 points) Determine whether the spring modeled by $mx'' + bx' + kx = 0$ with $m = 1\,\text{kg}$, $k = 3\,\text{N/m}$ and $b = 4\,\text{Ns/m}$ exhibits oscillations or not. (*No credit for just saying "yes" or "no."*)

2. (15 points) Find the terms up to t^3 in a power series expansion for a solution of

$$x''' - 3x'' + 3x' - x = 0, \qquad x(0) = 0,\ x'(0) = 0,\ x''(0) = 2.$$

3. (10 points) Solve the initial-value problem $D\vec{x} = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \vec{x}, \qquad \vec{x}(0) = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}.$

4. (15 points) Solve $D\vec{x} = A\vec{x}$, where $A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix}.$

5. (5 points) Find **one** nonzero solution of $D\vec{x} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{pmatrix} \vec{x}.$

(Yes, just one solution is good enough, so long as it is not the zero function.)

6. (15 points) Solve

$$(D-1)x = \begin{cases} 0 & t < 2 \\ 3 & t \geq 2 \end{cases}, \qquad x(0) = 1.$$

7. (5 points) Check the vectors $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 4 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ for linear independence.

Examination continues on next page

8. (5 points, *no credit unless every answer is correct*) For each of the following vectors decide whether it is an eigenvector of $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 9 \end{pmatrix}$, and if so, provide the corresponding eigenvalue. For each part, your answer should be either "NO" or a number; please put all your answers on the inside front blue cover of your examination booklet.

a. $\begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix}$ **b.** $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$ **c.** $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ **d.** $\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}$ **e.** $\begin{pmatrix} -2 \\ 0 \\ 2 \end{pmatrix}$ **f.** $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ **g.** $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$

h. $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ **i.** $\begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ **j.** $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ **k.** $\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$ **l.** $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ **m.** $\begin{pmatrix} 0 \\ 2 \\ 2 \end{pmatrix}$ **n.** $\begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

9. (5 points) Determine whether

$$\begin{aligned} x_1(t) &= 3c_1e^{4t} + c_2e^{-4t} \\ x_2(t) &= \ \ c_1e^{4t} + c_2e^{-4t} \end{aligned}$$

describes the general solution of the system

$$\begin{aligned} x_1' &= 5x_1 - 3x_2, \\ x_2' &= 3x_1 - 5x_2. \end{aligned}$$

10. (5 points, no partial credit) The matrix $\begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$ has a triple eigenvalue 1. Find 3 linearly independent generalized eigenvectors (you do not have to verify that they are linearly independent).

END OF EXAMINATION