

**Instructions:** No calculators, notes or books are allowed. Show all work to receive full credit. Remember to sign your blue book, indicating that you have neither given nor received assistance on this exam.

1. (20 points) Find the general solution to the differential equation

$$t \frac{dx}{dt} + 100x = 111t^{11}$$

for  $t > 0$ .

2. (20 points) Consider the ordinary differential equation

$$\frac{dx}{dt} = (1-x)(x-2)^2(x-3).$$

- (a) Find the location(s) of all equilibria.  
 (b) Make a phase portrait for the equation.  
 (c) Suppose that  $x(t)$  is a solution that satisfies  $x(0) = \frac{3}{2}$ . What is  $\lim_{t \rightarrow \infty} x(t)$ ?
3. (20 points) For which of the following initial conditions does the Existence and Uniqueness Theorem guarantee a unique solution to the ODE

$$\frac{dx}{dt} = \frac{x^{1/3}}{t-1}?$$

- (a)  $x(1) = 0$   
 (b)  $x(0) = 1$   
 (c)  $x(1) = 8$   
 (d)  $x(8) = 1$
4. (20 points) Determine whether or not the following functions are linearly independent on the interval  $-\infty < t < +\infty$ :

$$h_1(t) = t + e^t$$

$$h_2(t) = t - e^t$$

$$h_3(t) = e^t$$

Justify your answer!

5. (20 points) This problem concerns the differential equation

$$(t^2 D^2 - 2)x = 0.$$

on the interval  $0 < t < \infty$ .

- (a) What is the order of this equation? Is this equation linear? Does it have constant coefficients? Is it normal on the specified interval? Give reasons for your answers.  
 (b) Show that the functions

$$h_1(t) = t^{-1}$$

$$h_2(t) = t^2$$

solve the equation.

- (c) Do  $h_1(t)$  and  $h_2(t)$  generate the general solution on the specified interval? Justify your answer.