Tufts University

MATH 51	Department of Mathematics	Monday, Oct. 12, 2015
Differential Equations	Exam 1	12:00pm – 1:20pm

No calculators, books or notes are allowed on the exam. All electronic devices must be turned off and put away. **You must show all your work** in the blue book in order to receive full credit. *Please box your answers and cross out any work you do not want graded.* Make sure to sign your blue book. With your signature you are pledging that you have neither given nor received assistance on the exam. **There are two pages to this exam, totaling 6 questions.** *Good luck!*

1. Warm up:
   (a) (1 point each) Classify the following ODE's by order and whether it is linear or nonlinear. If the ODE is linear, also indicate if it is homogeneous or nonhomogeneous. Finally, if the ODE is linear, give the intervals where the ODE is normal.

   $$\text{(i)} \quad \left(\frac{d^3x}{dt^3}\right)^3 + \frac{dx}{dt} = 0$$

   $$\text{(ii)} \quad x\frac{dy}{dx} - y = \frac{\sqrt{1-e^{x/3}}}{8}$$

   $$\text{(iii)} \quad \frac{t}{t-1}\frac{d^2x}{dt^2} = \sqrt{t}x$$

   (b) (2 points) True or False: The function $g(t) = \sqrt{2t}$ is a solution to the ODE

   $$(t^{3/2}D^2 + \sqrt{t})x = \frac{4t-1}{\sqrt{8}}.$$

   (c) (2 points) True or False: The annihilator of $t^2\cos(t) + t^2$ is $(D^2+1)^3 + D^3$.
   (d) (2 points) True or False: Separation of variables can solve some nonlinear ODEs.

2. Consider the ODE

   $$(D^2 + 4)x = \cos(t). \tag{1}$$

   (a) (15 points) What is the general solution to equation (1)?
   (b) (6 points) What is the specific solution to equation (1) satisifying

   $$x(0) = x'(0) = 0?$$

3. (15 points) Solve the following differential equation using any method you see fit.

   $$x' + \cos(t)x = \cos(t)$$

4. (15 points) Are the functions $h_1(t) = t^2 + e^{t-1}$, $h_2(t) = 2t^2$, and $h_3(t) = 4e^t$ linearly independent? Justify your answer.

EXAM CONTINUES ON REVERSE

5. (20 points) Is the collection of solutions

$$x(t) = c_1 e^t + c_2 t e^t + c_3 (t-1) e^t + e^{-t}$$

the general solution to

$$(D^3 - 2D^2 + D)x = -4e^{-t}?$$

If not, give an initial condition of the form

$$\begin{aligned} x(t_0) &= \alpha_0 \\ x'(t_0) &= \alpha_1 \\ x''(t_0) &= \alpha_2 \end{aligned}$$

which cannot be satisfied by the proposed collection of solutions.

6. (20) Given that $h_1(t) = t^{-1}$ and $h_2(t) = t^2$ generate the complete solution to the ODE

$$(tD^2 - 2/t)x = 0, \quad t > 0,$$

what is the general solution to

$$(tD^2 - 2/t)x = t, \quad t > 0?$$