Name:____

Professors name:____

Due date: 10/2

1. Determine if the following series converge or diverge and explain why (i.e., what test you used). If the series converges, determine if it converges absolutely or conditionally.

   (a) $\sum_{n=2}^{\infty} \sqrt{\frac{n}{n^3+1}}$ (b) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\, 2^{3n}}{4^{2n+1}}$ (c) $\sum_{n=1}^{\infty} \frac{(-1)^n n^5}{5n^6-1}$

2. Let $f(x) = \ln(x) + 1$. In the following problem we will estimate $f(2)$ using linear approximation. We are NOT using Taylor polynomials to do this.

   (a) Give the linear approximation for $f(x)$ centered at 1. Call this linear approximation $L_1(x)$. Use $L_1(x)$ to estimate $f(2)$ and compare this with the value of $f(2)$ given by a calculator.

   (b) Graph $f(x)$ and $L_1(x)$ on the same axis and label Δx, Δy and dy on the graph. You may use the attached picture as a hint, but please redraw.

   (c) In the picture on the next page (Figure 1), there are two linear approximations to $f(x)$. One is the linear approximation centered at 1 labeled $L_1(x)$ and the other is centered at $\frac{3}{2}$ labeled $L_{\frac{3}{2}}(x)$. Let Δx_1 correspond to the changes in x from 1 to $\frac{3}{2}$, and Δx_2 correspond to the change in x from $\frac{3}{2}$ to 2. Let Δy_1 be the change in the value of $f(x)$ corresponding to Δx_1 and let Δy_2 be the change in the function $f(x)$ corresponding to Δx_2. Redraw (or print) the picture in Figure 1 on the next page and label Δx_1, Δx_2, Δy_1, and Δy_2.

   (d) Compute $L_1(\frac{3}{2}) - L_1(1)$ and $L_{\frac{3}{2}}(2) - L_{\frac{3}{2}}(\frac{3}{2})$ and label these on your redrawn graph from Part (c) with labels dy_1 and dy_2 respectively (Hint: One is B_1 and one is B_2). Explain why these are the appropriate labels for these values.

   (e) Which value from Part (d) is an estimate for Δy_1 and which is an estimate for Δy_2? Explain your answer.

   (f) Notice that $f(2) = f(1) + \Delta y_1 + \Delta y_2$. Use Part (e) to give an estimate for $f(2)$ and compare this to the value of $f(2)$ given by a calculator. Is this a better or worse estimate for $f(2)$ than in Part (a)?

   (g) In Parts (b)–(f) we estimated $f(2)$ using two linear approximations. Now estimate $f(2)$ using four linear approximations with the four centers 1, $\frac{5}{4}$, $\frac{3}{2}$ and $\frac{7}{4}$. There is no need to draw the graph, unless it helps for understanding.

Quiz continues on next page.

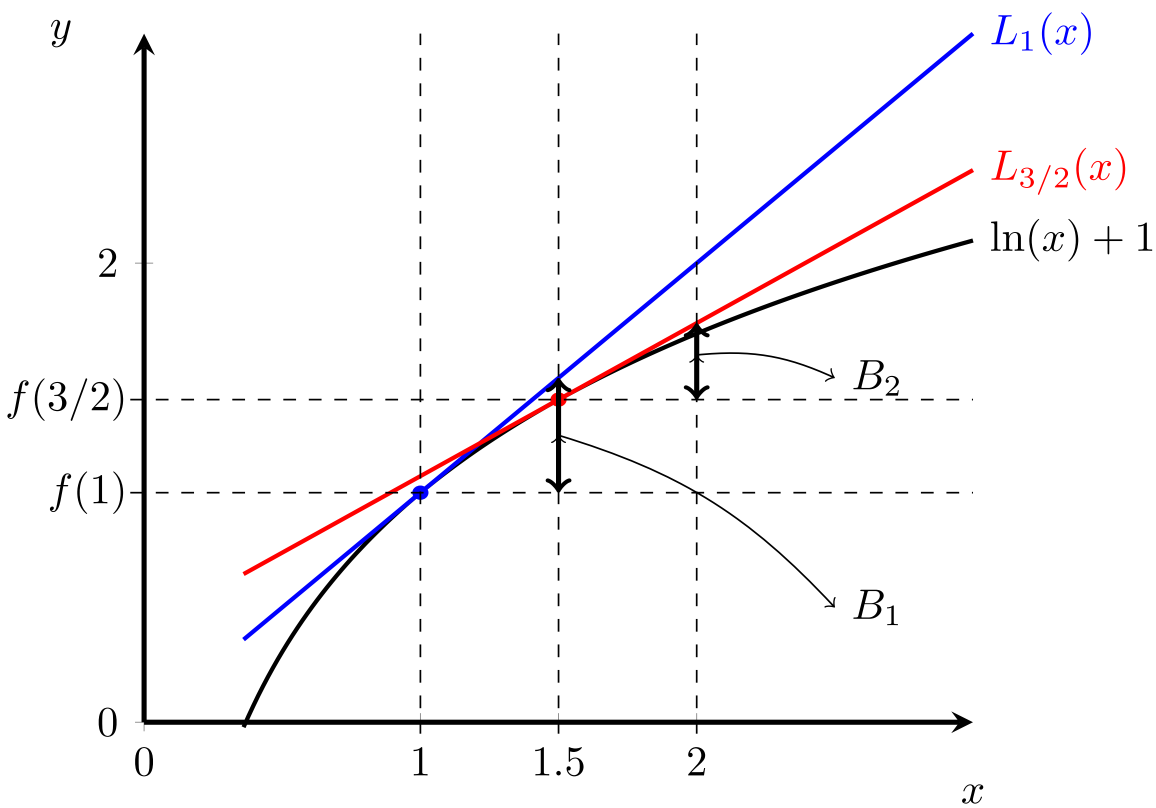

Figure 1: Two linear approximations of $\ln(x) + 1$ near $x = 2$.