Name:____

Professors name:____

Due date: 9/25

1. Determine if the following series converge or diverge and explain why.

   (a) $\sum_{n=1}^{\infty} \frac{n^3}{n!10^n}$ (b) $\sum_{n=3}^{\infty} \left(\frac{(\ln(n))^2}{n}\right)^n$ (c) $\sum_{n=1}^{\infty} \frac{n^5 - n^3 + 2}{5n^5 + n^4 - 10n}$

2. In this problem, we derive the formula:

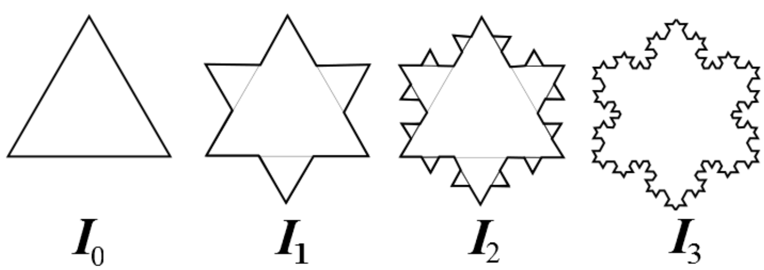
$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}.$$

   (a) Use the telescoping series method to compute $\sum_{i=1}^{n} \left[(1+i)^3 - i^3\right]$.

   (b) Expand (multiply out) the terms $\left[(1+i)^3 - i^3\right]$. Then use the fact that $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ to reduce $\sum_{i=1}^{n} \left[(1+i)^3 - i^3\right]$ so that the only summations you have left are $\sum_{i=1}^{n} i^2$.

   (c) Set the results of part (a) to equal the results from part (b) and solve for $\sum_{i=1}^{n} i^2$.

3. The fractal snowflake is constructed as follows: Let I_0 be an equilateral triangle with area 1 and have sides of length ℓ. The figure I_1 is obtained by replacing the middle third of each side of I_0 by a new outward equilateral triangle with sides of length $\frac{\ell}{3}$ (see figure). The process is repeated where I_{n+1} is obtained by replacing the middle third of each side of I_n by a new outward equilateral triangle with sides of length $\frac{\ell}{3^{n+1}}$. The limiting figure as $n \longrightarrow \infty$ is called the fractal snowflake.



   a) Let L_n be the perimeter of I_n. So $L_0 = 3\ell$. Show that the $\lim_{n\to\infty} L_n = \infty$

   b) Let A_n be the area of I_n. So $A_0 = 1$. Find the $\lim_{n\to\infty} A_n$. It exists.

Quiz continues on next page.

This shape is called the Koch snowflake or Koch Island. Here are some fun Vi Hart videos on it!

- Self-similarity of fractals: https://www.youtube.com/watch?v=dsvLLKQCxeA
- Fractional dimension: https://www.youtube.com/watch?v=Oc8sWN_jNF4

Fractal shapes like these are often found in nature because they are impressively efficient. For example, the alveoli lining our lungs have a fractal-like shape. Greater surface area of the lining of our lungs means greater exchange of oxygen with the body. So, much like the Koch snowflake has an arbitrarily large boundary length and a finite area interior, the surface area of our lungs is very large, even though the volume of our lungs is compact. In fact, the surface area of human lungs is about the size of a tennis court!

Read more about fractals and human biology here:

http://www.fractal.org/Life-Science-Technology/Publications/
Fractals-and-Human-Biology.pdf