You are responsible for all material covered in class, on the homework, and on all quizzes. One question on the midterm may come from a written quiz, so study those. Below is a study guide.

L'Hopital's Rule

- Know how to apply L'Hopitals rule in normal and in indeterminate forms.
- Examples: Be able to do the following problems:
  1. $\lim_{x\to 0} \frac{\sin(x)}{x}$.
  2. $\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right)$.
  3. $\lim_{x\to\infty} (x - \sqrt{x^2-3x})$.
  4. $\lim_{x\to 0} x^x$.
  5. $\lim_{x\to\infty} \left(1+\frac{a}{x}\right)^x$ for a constant a.

Sequences

- Know the definition of a sequence.
- Know what it means for a sequence to converge.
- Know how to use functions to show the convergence of a sequence (p. 607).
- Know the properties of limits (p. 607).
- Know the definitions for increasing, decreasing, strictly increasing, strictly decreasing, monotonic and bounded sequences.
- Know what a geometric sequence is and when it converges/diverges.
- Know the statement of the squeeze theorem.
- Know that bounded monotonic sequences converge.
- Review problems from Section 8.2.

Series

- Know the definition of an infinite series.
- Know what the sequence of partial sums for a series is.
- Know what it means for an infinite series to converge.
- Know what a geometric series is and when it converges/diverges.
- Know how to use the geometric series to change a repeating decimal into a fraction (see Example 2 on p. 622).
- Know what a p-series is and for what value of p it converges.
- Know what an alternating series is and under what conditions it converges.
- Know how to handle telescoping series, for example:
  1. $\sum_{k=1} \left(\frac{1}{3^k} - \frac{1}{3^{k+1}}\right)$,
  2. $\sum_{k=1} \ln\left(\frac{k+1}{k+3}\right)$.
- Know properties of convergent series (p. 636).
- Know the statement of and how to apply the divergence test.
- Know the statement of and how to apply the ratio test **(NOTE: THIS IS A STRONGER VERSION THEN THE RATIO TEST IN THE BOOK)**.

  Let $\sum a_k$ be a series and $r = \lim_{k\to\infty} \left|\frac{a_{k+1}}{a_k}\right|$.
  1. If $0 \le r < 1$, then the series converges.
  2. If $r > 1$ (including $= \infty$) then the series diverges.
  3. If $r = 1$ the test is inconclusive.
- Know the statement of and how to apply the root test **(NOTE: THIS IS A STRONGER VERSION THEN THE ROOT TEST IN THE BOOK)**.

  Let $\sum a_k$ be a series and $r = \lim_{k\to\infty} \sqrt[k]{|a_k|}$.
  1. If $0 \le r < 1$, then the series converges.
  2. If $r > 1$ (including $= \infty$) then the series diverges.
  3. If $r = 1$ the test is inconclusive.
- Know the statement of and how to apply the comparison and limit comparison test.
- We wont give a list of problems to practice, Chapter 8 is full of excellent examples (try the odd version of the assigned homework problems).

Linear Approximation

- Know how to find the linear approximation of a function at a point.
- Know under what conditions a linear approximation is a "good" approximation for a function.
- For example, use linear approximations to estimate the values of:

  1. $\sqrt{1.003}$
  2. $\tan^{-1}(1.001)$

Newton's Method

- Know what the sequence of Newton iterations is for a given function (See Section 4.8, particularly Figure 4.86).
- Know how the method is derived using linear approximations.
- Be able to explain with the use of a graph, whether using a given starting guess will lead to a convergent or divergent sequence.

Taylor polynomials/series

- Know the definition of and how to compute an n-th degree Taylor polynomial (p. 664).
- Look over example 2 on p. 664.
- Know how to find bounds for the error terms of Taylor polynomials (p. 668-669).
- Look over problems in Section 9.1.
- Examples: Give the Taylor polynomials for the following.

  1. $f(x) = e^x$ centered at 0 of degree n.
  2. $f(x) = \ln(x)$ centered at 1 of degree n.
  3. $f(x) = \sin(x)$ centered at π of degree 5 and of degree 6.

Taylor/Power Series:

- Know what a Taylor and Maclaurin series are.
- Taylor Series you should know, with the radii of convergence:
  - e^x centered at 0
  - $\ln(1-x)$ centered at 0
  - $\sin(x)$ and $\cos(x)$ centered at 0
  - $\ln(x)$ centered at 1
  - $\dfrac{1}{1-x}$ centered at 0
- Know how to manipulate power series representations of functions to obtain new power series representations of functions with the interval of convergence. For example, find power series representations for:
  - $f(x) = g(3x^3)$ where g is a function listed above.
  - $f(x) = \ln(1+x^2)$ centered at zero.
  - $f(x) = \arctan(x)$ centered at zero.
- Know how to find functions represented by a given power series. Specific examples:
  - Find the function represented by $\sum_{k=1}^{\infty} kx^k$ by differentiating a power series you know. What is the interval on which the power series converges to the function?
  - Show $\int e^x dx = e^x + C$ using power series.

Riemann Sums:

- Definition of the Riemann sum.
- How to compute Riemann sums using left and right endpoints.
- Example: Compute Riemann sum using left endpoints of $f(x) = x^2$ from 1 to 3 and $n = 3$.
- How increasing functions v.s. decreasing effect whether left/right endpoints give an over or underestimate.
- See Chapter 5 on Riemann Sums using Sigma Notation for formulas for computing sums.

Fundamental Theorem of Calculus:

- Know the definition of the Area Function.
- Know the definition and implications of the Fundamental Theorem Part I:

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x).$$

- Know the definition and implications of the Fundamental Theorem Part II:

$$\int_a^b f(x)\,dx = f(b) - f(a).$$

Apps of Riemann Sums/Integrals:

- Given a continuous velocity function, compute distance traveled over a given time frame. Same for acceleration, population growth rates, and other application problems from Section 6.1.
- Given discrete data points for velocity, approximate the distance traveled (same for other applications).
- Area between curves
- u-substitution.

Computing Volumes:

- Understand how to find the volume of various shapes using cross-sectional areas.
- Examples:
  - Volume of a square cross sections on circle base
  - Volume of pyramid
  - Volume of equilateral triangles on circle base (gnome/elf hat)
- Volumes by rotation. Using integration with respect to x and y.
- Disk, Washer, and Shell Methods.
- Examples:
  - The region bounded by $f(x) = \sqrt{x}$, $y = 0$ and $x = 3$ rotated about x-axis.
  - The region bounded by $f(x) = \sqrt{x}$, $y = -1$ and $x = 3$ rotated about $y = -1$.
  - The region bounded by $f(x) = \sqrt{x}$, $y = 1$ and $x = 3$ rotated about $y = 1$.
  - The region bounded by $f(x) = \sqrt{x}$, $y = 0$ and $x = 3$ rotated about $y = -1$.
  - The region bounded by $f(x) = \sqrt{x}$, $y = -1$ and $x = 3$ rotated about $y = -1$.
  - The region bounded by $y = x^2$, $y = 9x^2$ and $y = 4$ rotated about $x = 6$.
  - The region bounded by $f(x) = \sqrt{x}$, $y = 0$, and $x = 3$ rotated about $y = 5$.
  - The region bounded by $y = |x|$, $x = -(y+1)^2$, $y = -2$ and $y = 3$ rotated about $x = 4$.

Mass/Work/Force:

- Know how to compute the mass of objects with a density function in one variable.
- Know how to compute the work required to lift water out of a tank (in terms of density, volume, gravity and distance). There are many examples in the book.
- Know how to compute the work required to stretch or compress a spring with given spring constant.
- Know how to compute the force caused by hydrostatic pressure (see the dam examples).

Arc length:

- Know how to apply the arc length formula.
- Examples: Find the arc length of the following functions on the interval given

  a) $f(x) = (x+1)^{\frac{3}{2}}$ from -1 to 1

  b) $f(x) = \dfrac{e^x + e^{-x}}{2}$ from 0 to $\ln(2)$

  c) $f(x) = \sqrt{1-x^2}$ from -1 to 1

Integration by Parts:

- Formula for integration by parts page 516 and 519.
- $\int x \sin(x)\, dx$
- $\int x^2 \sin(x)\, dx$
- $\int e^x \sin(x)\, dx$
- $\int \ln x\, dx$
- $\int x^3 \sin(x^2)\, dx$

Trigonometric Integrals

- Know the techniques for powers and products of sin and cos.
- Know the techniques for powers and products of tan and sec.
- Know the techniques for powers and products of cot and csc.
- Know half/double angle identities
- It might be useful to know reduction formulas.
- Know $\displaystyle\int \sec(x)\, dx$

- Examples:

  a) $\int \tan(x)\ dx$

  b) $\int \sin^3(x)\cos^6(x)\ dx$

  c) $\int \tan^3(x)\sec^5(x)\ dx$

  d) $\int \tan^2(x)\ dx$

  e) $\int \sin^2(x)\cos^4(x)\ dx$

  f) $\int \sin^2(x)\cos^4(x)\ dx$

  g) $\int \tan(x)\csc(x)\ dx$

Trigonometric Substitution

- Know how to recognize and apply trig substitutions.
- Know the "triangle" method for simplifying trig compositions.
- Examples:

  a) $\int \sqrt{2-3x^2}\ dx$

  b) $\int \frac{dx}{x^2\sqrt{1+x^2}}\ dx$

  c) $\int \frac{dx}{(1-x^2)^{\frac{3}{2}}}\ dx$

Partial Fractions

- Use partial fractions to compute an integral.
- Know how to complete a square and use this in integration.
- Integrate the following:

  1. $\int \frac{x^2}{x^3-16x}\ dx$
  2. $\int \frac{x^2}{(x^2+1)(x+1)}\ dx$
  3. $\int \frac{x^3}{(x^2+1)(x+1)}\ dx$
  4. $\int \frac{x^3+1}{x^2-1}\ dx$

- Other examples at the end of 7.5 (p. 549).
- Know how to apply the method of partial fractions to telescoping series.

$$-\sum_{k=2}^{1000} \frac{1}{x^2+7x+12}$$

Differential equations

- Be able to recognize a differential equation and how to interpret the meaning.
- Be able to do simple (separable) differential equations.
- For practice try problems in Section 7.9.

Improper Integrals

- Recognize and set up the correct limits for improper integrals.
- Check that the function to be integrated is defined on the entire domain of integration.
- Examples:

  a) $\int_0^1 \ln(x)\ dx$

  b) $\int_{-1}^1 \frac{1}{x^2}\ dx$

  c) $\int_0^\infty 2^{-x}\ dx$

  d) $\int_{-\infty}^0 e^x\ dx$

Parametric Equations

- Sketch parametric curves.
- Describe a curve using parametric equations.
- Converting parametric curves to standard functions (when possible) and vice-versa.
- Find the slope of a tangent line to a parametric curve.
- Examples: Graph the following parametric curves for different intervals of t.

  a) $r(t) = \langle a\cos(kt), a\sin(kt)\rangle$ for $a, k \in \mathbb{R}$

  b) $r(t) = \langle t\cos(2\pi t), t\sin(2\pi t)\rangle$

  c) $r(t) = \langle 1+\cos(2\pi t), -2+\sin(2\pi t)\rangle$

  d) $r(t) = \left\langle t, t^2\right\rangle$

  e) $r(t) = \left\langle t^2, t\right\rangle$

  f) $r(t) = \left\langle t^3, t^2\right\rangle$

- Give a parametrization for the line segment between $(1, 2)$ and $(3, 10)$, make sure to state the interval for t.
- Find the slope of the tangent line to the parametric curve given by $x(t) = \frac{1}{t}$, $y(t) = t^2$ at the point $\left(\frac{1}{2}, 4\right)$.

Polar Coordinates

- Know what polar coordinates are.
- Sketch curves given in polar coordinates.
- Know how to graph polar functions especially:
  a) Basic circles (see p. 723)
  b) Cardioids
  c) "polar flowers"
- Know how to graph polar coordinates. For example:
  - $r(\theta) = 1 + sin(\theta)$
  - $r = 3\sin(2\theta)$
- Know how to compute areas in polar coordinates. For example:
  - The area inside the 4 leaf rose given by $r = 2\cos(2\theta)$.
  - The area inside the circle $r = 1$ and the curve $r = 1 + \cos(\theta)$.
  - the area inside $r = 1+\cos(\theta)$ and outside the circle $r = 1$.
- Review the goat problem.