Integrals involving Powers of Trigonometric Functions

Derivatives

$(\sin x)' = \cos x$ $\qquad$ $(\tan x)' = \sec^2 x$ $\qquad$ $(\cot x)' = -\csc^2 x$

$(\cos x)' = -\sin x$ $\qquad$ $(\sec x)' = \sec x \tan x$ $\qquad$ $(\csc x)' = -\csc x \cot x$

Antiderivatives

$\int \tan x \, dx = \ln|\sec x| + C$ $\qquad$ $\int \cot x \, dx = \ln|\sin x| + C$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\qquad$ $\tan^2 x + 1 = \sec^2 x$ $\qquad$ $1 + \cot^2 x = \csc^2 x$

$\int \sin^m x \cos^n x \, dx$

1. If m or n is odd, split off a single power of $\sin x$ or $\cos x$ from the function whose power is odd to serve as $\pm du$. Convert the even powers that are left to the cofunction using the Pythagorean identity.
2. If both m and n are even, reduce powers down to the first power by successively using the double angle identities as many times as needed:

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$\int \tan^m x \sec^n x \, dx$

1. (a) If n is even split off $\sec^2 x$ to serve as du and convert everything else to powers of tangent using the Pythagorean identity if necessary. Let $u = \tan x$.

   (b) If m is odd and $n \geq 1$ split off $\sec x \tan x$ to serve as du and convert everything else to powers of secant using the Pythagorean identity if necessary. Let $u = \sec x$.

   (c) If m is even and n is odd, convert the powers of tangent to powers of secant to produce a polynomial in $\sec x$ and apply reduction formula 4 to each term.

   (d) Given just $\int \tan^n x \, dx, n > 1$, split off $\tan^2 x$, replace with $\sec^2 x - 1$. Distribute over this difference and break into separate integrals. Repeat if necessary.