Math 34 Quiz 6b Fall 2013

Name KEY Section ______

Determine whether the following series converges or diverges. Justify your answer. State and check hypotheses of any test, rules or theorem you use.

$\sum_{n=2}^{\infty} \frac{1}{n\sqrt{\ln n}}$ Integral Test Let $f(x) = \frac{1}{x\sqrt{\ln x}}$, $x \geq 2$

Then ① $f(x)$ is continuous and positive for $x \geq 2$.

② Need to show f is eventually decreasing

Option 1 $f(x) = (x\sqrt{\ln x})^{-1}$

$$f'(x) = -1 \underbrace{(x\sqrt{\ln x})^{-2}}_{+} \underbrace{\left[1 \cdot \sqrt{\ln x} + x \cdot \tfrac{1}{2}(\ln x)^{-\frac{1}{2}} \cdot \tfrac{1}{x}\right]}_{+} < 0 \text{ for } x \geq 2$$

Since $f'(x) < 0$ for $x \geq 2$, f is decreasing for $x \geq 2$

Option 2 Since x, $\sqrt{\ln x}$ are both positive and increasing $x \geq 2$, $x\sqrt{\ln x}$ is increasing $x \geq 2$, so $\frac{1}{x\sqrt{\ln x}}$ is decreasing for $x \geq 2$.

So we can use the integral test.

$$\int_2^{\infty} \frac{dx}{x\sqrt{\ln x}} = \lim_{t\to\infty} \int_2^t \frac{dx}{x\sqrt{\ln x}} = \lim_{t\to\infty} 2\sqrt{\ln x}\Big|_2^t = \lim_{t\to\infty} \underbrace{2\sqrt{\ln t}}_{\to\infty} - 2\sqrt{\ln 2}$$

$$\left[\text{"form" } \int \frac{du}{\sqrt{u}} = \int u^{-\frac{1}{2}}\,du = 2\sqrt{u} + C = 2\sqrt{\ln x} + C\right]$$

Since the integral diverges, the series diverges.