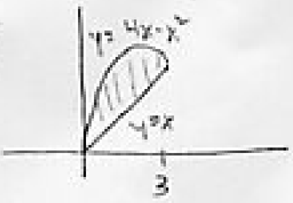
Math 12 Exam I Solutions Fall 2011

$\boxed{1}$ (12)

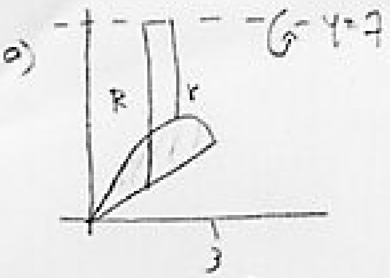


$4x - x^2 = x$

$3x - x^2 = 0$

$x(3-x) = 0, \quad x = 0, 3$

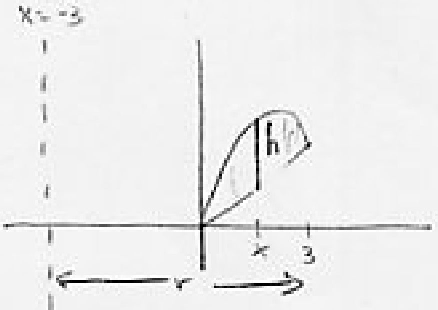
a)



Washers $V = \pi \int_0^3 (R^2 - r^2)\,dx$

$= \pi \int_0^3 (7-x)^2 - (7-(4x-x^2))^2\,dx$

b)



Shells $V = 2\pi \int_0^3 rh\,dx$

$= 2\pi \int_0^3 (x - -3)((4x-x^2)-x)\,dx$

$= 2\pi \int_0^3 (x+3)((4x-x^2)-x)\,dx$

$\boxed{2}$ (8) $\frac{x^2-2x+1}{x^4(x^2+9)^2(x-7)(x+1)}$

$= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{Ex+F}{x^2+9} + \frac{Gx+H}{(x^2+9)^2} + \frac{I}{(x-7)} + \frac{J}{x+1}$

$\boxed{3}$ (10) $\int \cos^3 x\,dx = \int \cos^2 x \cos x\,dx = \int (1-\sin^2 x)\cos x\,dx = \int (1-u^2)\,du = u - \frac{u^3}{3} + C$

$u = \sin x \quad du = \cos x\,dx$

$= \boxed{\sin x - \frac{\sin^3 x}{3} + C}$

$\boxed{4}$ (10) $\int \ln(2t)\,dt = t\ln(2t) - \int dt = \boxed{t\ln(2t) - t + C}$

parts $u = \ln(2t) \quad dv = dt$

$du = \frac{2}{2t}dt \quad v = t$

$\boxed{5}$ (10) $\int \frac{dx}{\sqrt{4+9x^2}} = \int \frac{2/3 \sec^2\theta\,d\theta}{2\sec\theta} = \frac{2}{3}\cdot\frac{1}{2}\int \sec\theta\,d\theta$

$= \frac{1}{3}\ln|\sec\theta + \tan\theta| + C$

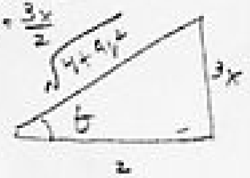
$x = \frac{2}{3}\tan\theta$

$dx = 2/3 \sec^2\theta\,d\theta$

$\sqrt{4+9x^2} = \sqrt{4+9(4/9\tan^2\theta)}$

$= \sqrt{4(1+\tan^2\theta)} = 2\sec\theta$

$\tan\theta = \frac{3x}{2}$



$= \boxed{\frac{1}{3}\ln\left|\frac{\sqrt{4+9x^2}}{2} + \frac{3x}{2}\right| + C}$

$\boxed{6}$ (10) $\int_0^2 \sqrt{16-x^2}\,dx = 16\int_0^{\pi/6} \cos^2\theta\,d\theta = 8\int_0^{\pi/6} (1+\cos 2\theta)\,d\theta = 8\left[\theta + \frac{1}{2}\sin 2\theta\right]\Big|_0^{\pi/6}$

$x = 4\sin\theta$

$dx = 4\cos\theta\,d\theta$

$\sqrt{16-x^2} = \sqrt{16-16\sin^2\theta} = \sqrt{16\cos^2\theta}$

$= 4\cos\theta$

Limits

$x=2 \quad 2 = 4\sin\theta, \ \sin\theta = 1/2, \ \theta = \pi/6$

$x=0 \quad 0 = 4\sin\theta \quad \sin\theta = 0 \quad \theta = 0$

$= 8\left[\left(\pi/6 + \frac{1}{2}\sin\pi/3\right) - \left(0 + \frac{1}{2}\sin 0\right)\right]$

$= 8\left(\pi/6 + \frac{1}{2}\left(\sqrt{3}/2\right)\right)$

$= \frac{4\pi}{2} + 2\sqrt{3}$

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$\boxed{7}$ (10) $\int_0^1 y e^{-2y}\,dy = -\frac{1}{2} y e^{-2y}\Big|_0^1 + \frac{1}{2}\int e^{-2y}\,dy$

Parts

$\left(\begin{array}{ll} u = y & dv = e^{-2y}dy \\ du = dy & v = -\frac{1}{2}e^{-2y} \end{array}\right) = -\frac{1}{2} y e^{-2y}\Big|_0^1 + \frac{1}{2}\left(-\frac{1}{2}\right)e^{-2y}\Big|_0^1$

$= -\frac{1}{2}\left[1 \cdot e^{-2} - 0\right] - \frac{1}{4}\left[e^{-2} - e^{0}\right] = -\frac{1}{2}e^{-2} - \frac{1}{4}e^{-2} + \frac{1}{4}$

$= \frac{1}{4} - \frac{3}{4}e^{-2}$

$\boxed{8}$ (10) $\int \frac{x^2+1}{x^2-x}\,dx = \int\left(1 + \frac{x+1}{x^2-x}\right)dx = \int\left(1 - \frac{1}{x} + \frac{2}{x-1}\right)dx$

(divide: $x^2 - x \overline{\smash{)}\, x^2 + 0 \cdot x + 1}$ with quotient 1; $-(x^2 - x)$; remainder $x+1$)

[PF's
$\frac{x+1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$

$x+1 = A(x-1) + Bx$

$x=0 \quad 1 = A(-1)$

$x=1 \quad 2 = B$]

$= x - \ln|x| + 2\ln|x-1| + C$

$\boxed{19}$ $\int_{-1}^{0}\frac{dx}{(x+1)^{1/3}} = \lim_{t\to -1^+}\int_t^0 (x+1)^{-1/3}dx = \lim_{t\to -1^+} -3(x+1)^{-1/3}\Big|_t^0 = \lim_{t\to -1^+} \frac{-3}{(x+1)^{1/3}}\Big|_t^0$

$(-1, 0)$

$= \lim_{t\to -1^+}\left[\frac{-3}{1} + \underbrace{\frac{3}{(t+1)^{1/3}}}_{\infty}\right]$ Integral Diverges

$\boxed{10}$ $\int_3^{\infty}\frac{e^{1/x}}{x^2}\,dx = \lim_{t\to\infty}\int_3^t \frac{e^{1/x}}{x^2}\,dx = \lim_{t\to\infty}\left(-e^{1/x}\right)\Big|_3^t = \lim_{t\to\infty}\left[\underbrace{-e^{\overset{\to 0}{1/t}}}_{-1} + e^{1/3}\right]$

$-1 + e^{1/3}$

$\left[\begin{array}{ll} \int \frac{e^{1/x}}{x^2}\,dx & = -\int e^u\,du \\ u = 1/x & = -e^u + C \\ du = -1/x^2\,dx & = -e^{1/x} + C \\ -du = dx & \end{array}\right]$

Converges to $\boxed{e^{1/3} - 1}$