Math 12 Exam I Solutions Fall 2010

$\boxed{1}$ (6) c) $\boxed{x = \frac{1}{\sqrt{3}} \sec\theta}$ b) $x^2+6x+25 = x^2+6x+9-9+25 = (x+3)^2+16$ $\boxed{x+3=4\tan\theta}$

$\boxed{2}$ (10) $\frac{16}{x(x^2+4)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}$

⊛ $16 = A(x^2+4)^2 + \underbrace{(Bx+C)x(x^2+4)}_{(Bx+c)(x^3+4x)} + (Dx+E)x$

$x=0$ $16 = A(16) \rightarrow \boxed{A=1}$

$16 = x^4+8x^2+16 + Bx^4+Cx^3+4Bx^2+4Cx + Dx^2+Ex = (1+B)x^4 + Cx^3 + (8+4B+D)x^2 + (4C+E)x + 16$

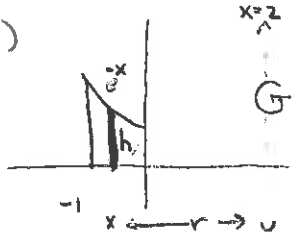
x^4 : $0 = 1+B \rightarrow \boxed{B=-1}$

x^3 : $\boxed{0=C}$

x^2 : $0 = 8+4B+D = 8-4+D \rightarrow \boxed{D=-4}$

x : $0 = 4C+E = 0+E \rightarrow \boxed{E=0}$

$\boxed{3}$ (12)



$$V = 2\pi \int_{-1}^{0} rh\,dx = 2\pi \int_{-1}^{0} (2-x)e^{-x}\,dx = 2\pi\left[-e^{-x}(2-x)\Big|_{-1}^{0} - \int_{-1}^{0} e^{-x}\,dx\right]$$

(parts: $u=2-x$ $dv=e^{-x}$
$du=-dx$ $v=-e^{-x}$)

$$= 2\pi\left[\left(-e^{0}(2)+e^{+1}(2--1)\right) + e^{-x}\Big|_{-1}^{0}\right]$$

$$= 2\pi\left[-2+3e+(e^{0}-e^{1})\right] = 2\pi\left[-2+3e+1-e\right] = \boxed{2\pi\left[2e-1\right]}$$

$\boxed{4}$ (12) $\int_{-1}^{1} \frac{2\,dx}{x^2-2x} = \int_{-1}^{1} \frac{2\,dx}{x(x-2)} = \int_{-1}^{0} \frac{2\,dx}{x(x-2)} + \int_{0}^{1} \frac{2\,dx}{x(x-2)}$

PF's $\frac{2}{x(x-2)} = \frac{A}{x} + \frac{B}{x-2}$

⊛ $2 = A(x-2) + Bx$

$x=0$ $2=-2A \rightarrow \boxed{A=-1}$

$x=2$ $2=2B \rightarrow \boxed{B=1}$

$$\int_{0}^{1} \frac{2\,dx}{x(x-2)} = \lim_{t\to0^+} \int_{t}^{1} \frac{2\,dx}{x(x-2)} = \lim_{t\to0^+} \int_{t}^{1} \left(\frac{1}{x-2} - \frac{1}{x}\right)dx$$

$$= \lim_{t\to0^+} \left[\ln|x-2| - \ln|x|\right]\Big|_{t}^{1} = \lim_{t\to0^+}\left[\left(\underset{0}{\ln|-1|} - \underset{0}{\ln|1|}\right) - \left[\underbrace{\ln|t-2|}_{\ln 2} - \underbrace{\ln|t|}_{-\infty}\right]\right]$$

The integral <u>diverges</u>.

$\boxed{5}$ (10) $\frac{dy}{dx} = \frac{4+x}{yx}$, $x > \frac{1}{2}$, $y(1) = -3$

$y\,dy = \frac{4+x}{x}\,dx$

$\int y\,dy = \int\left(\frac{4}{x}+1\right)dx \to \frac{y^2}{2} = 4\ln|x| + x + c$ ↲ rename c

$y^2 = 8\ln|x| + 2x + c$

$y = \pm\sqrt{8\ln|x| + 2x + c}$

Solve for c: $-3 = -\sqrt{8\underset{0}{\ln|1|} + 2 + c} \to 9 = 2+c \Rightarrow c = 7$

$\boxed{y = -\sqrt{8\ln|x| + 2x + 7}}$

$\boxed{6}$ (10) $\int\sqrt{9-x^2}\,dx = 9\int\cos^2\theta\,d\theta = \frac{9}{2}\int(1+\cos 2\theta)\,d\theta = \frac{9}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right] + C$

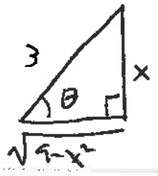
$x = 3\sin\theta$, $-\frac{\pi}{2} \le \theta \le \pi/2$

$dx = 3\cos\theta\,d\theta$

$\sqrt{9-9\sin^2\theta} = \sqrt{9\cos^2\theta} = |3\cos\theta| = 3\cos\theta$

$= \frac{9}{2}\left[\theta + \frac{1}{2}(2\sin\theta\cos\theta)\right] + C = \frac{9}{2}\left[\theta + \sin\theta\cos\theta\right] + C$

$\sin\theta = \frac{x}{3}$



$= \boxed{\frac{9}{2}\left[\sin^{-1}(x/3) + \frac{x\sqrt{9-x^2}}{9}\right] + c}$

$\boxed{7}$ $\int_0^4 \frac{4x+5}{x^2+16}\,dx = \int_0^4 \frac{4x\,dx}{x^2+16} + \int_0^4 \frac{5\,dx}{x^2+16} = 2\ln|x^2+16|\Big|_0^4 + \frac{5}{4}\tan^{-1}\frac{x}{4}\Big|_0^4$

$= 2\left[\ln|32| - \ln|16|\right] + \frac{5}{4}\left[\tan^{-1}1 - \tan^{-1}0\right] = 2\ln 2 + \frac{5}{4}\left[\frac{\pi}{4} - 0\right] = 2\ln 2 + \frac{5\pi}{16} = \boxed{\ln 4 + \frac{5\pi}{16}}$

$\boxed{8}$ $\int \tan^2(2x)\sec^4(2x)\,dx = \int \tan^2(2x)\overbrace{\sec^2(2x)}^{1+\tan^2(2x)}\sec^2(2x)\,dx = \frac{1}{2}\int u^2(1+u^2)\,du = \frac{1}{2}\int(u^2+u^4)\,du$

$\left(\begin{array}{l} u = \tan 2x \\ du = 2\sec^2 2x\,dx \\ \frac{1}{2}du = \sec^2 2x\,dx \end{array}\right)$

$= \frac{1}{2}\left(u^3/3 + u^5/5\right) + c$

$= \boxed{\frac{1}{2}\left(\frac{1}{3}\tan^3(2x) + \frac{1}{5}\tan^5(2x)\right) + C}$

$\boxed{9}$ $\int \sqrt[4]{\sin x}\,\cos^3 x\,dx = \int(\sin x)^{1/4}\cos^2 x\,\cos x\,dx = \int(\sin x)^{1/4}(1-\sin^2 x)\cos x\,dx$

$\left(\begin{array}{l} u = \sin x \\ du = \cos x\,dx \end{array}\right)$

$= \int u^{1/4}(1-u^2)\,du = \int\left(u^{1/4} - u^{9/4}\right)du = \frac{4}{5}u^{5/4} - \frac{4}{13}u^{13/4} + C$

$= \frac{4}{5}\sin^{5/4}x - \frac{4}{13}\sin^{13/4}x + C$

$\boxed{10}$ $\int \ln(x^2+1)\,dx = x\ln(x^2+1) - \int\frac{2x^2\,dx}{x^2+1} = x\ln(x^2+1) - \int\left(2 - \frac{2}{x^2+1}\right)dx$

parts: $u = \ln(x^2+1)$ $dv = dx$

$du = \frac{2x}{x^2+1}dx$ $v = x$

$$\begin{array}{r} 2 \\ x^2+1\,\big)\overline{\,2x^2+0\,} \\ -(2x^2+2) \\ \hline -2 \end{array}$$

$= \boxed{x\ln(x^2+1) - 2x + 2\tan^{-1}x + C}$