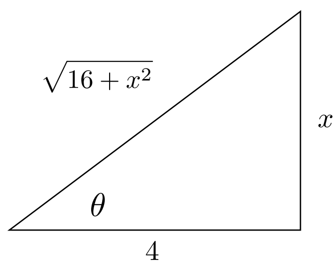
1. $$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos^2(6x)\ dx = \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{2}(1+\cos(12x))\ dx = \frac{1}{2}\Big[x + \frac{1}{12}\sin(12x)\Big]_{\pi/3}^{\pi/2}$$
$$= \frac{1}{2}\left(\frac{\pi}{2} + \frac{1}{12}\sin(6\pi) - \left(\frac{\pi}{3} + \frac{1}{12}\sin(4\pi)\right)\right) = \frac{\pi}{12}$$

2. $$\int x^3(16+x^2)^{3/2}\ dx$$

$x = 4\tan\theta \quad dx = 4\sec^2\theta\ d\theta$



From triangle we have
$\sqrt{16+x^2} = 4\sec\theta$ and substituting we get

$$\int 4^3\tan^3\theta \cdot 4^3\sec^3\theta \cdot 4\sec^2\theta\ d\theta = 4^7\int \tan^3\theta\sec^5\theta\ d\theta$$

$$= 4^7\int(\sec^2 - 1)\sec^4\theta \cdot \sec\theta\tan\theta\ d\theta$$

$u = \sec\theta$
$du = \sec\theta\tan\theta\ d\theta$

$$= 4^7\int(u^2-1)u^4\ du = 4^7\int(u^6-u^4)du = 4^7(\frac{1}{7}u^7 - \frac{1}{5}u^5) + C$$

$$= 4^7(\frac{(16+x^2)^{7/2}}{7\cdot 4^7} - \frac{(16+x^2)^{5/2}}{5\cdot 4^5}) + C$$

3. $\int e^x \tan^2(e^x)\sec^2(e^x)\ dx$

Let $u = e^x$ so $du = e^x dx$ and we get

$$\int \tan^2 u \sec^2 u\ du$$

Now let $v = \tan u$ so $dv = \sec^2 u\ du$ and the integral becomes

$$\int v^2\ dv = \frac{1}{3}v^3 + C = \frac{1}{3}\tan^3 u + C = \frac{1}{3}\tan^3(e^x) + C$$

4. $\dfrac{11x^2 - 3x - 2}{x(x^2-1)} = \dfrac{11x^2 - 3x - 2}{x(x-1)(x+1)} = \dfrac{A}{x} + \dfrac{B}{x-1} + \dfrac{C}{x+1}$

$11x^2 - 3x - 2 = A(x-1)(x+1) + Bx(x+1) + Cx(x-1)$

substitute $x = 0$: $\quad -2 = -A \quad \rightarrow A = 2$
substitute $x = 1$: $\quad 6 = 2B \quad \rightarrow B = 3$
substitute $x = -1$: $\quad 12 = 2C \quad \rightarrow C = 6$

$$\int \frac{11x^2 - 3x - 2}{x(x^2-1)} = \int (\frac{2}{x} + \frac{3}{x-1} + \frac{6}{x+1})\ dx = 2\ln|x| + 3\ln|x-1| + 6\ln|x+1| + C$$

5. $\int x^4 \ln(x)\ dx$

$u = \ln(x) \qquad dv = x^4$
$du = \dfrac{1}{x} \qquad v = \dfrac{1}{5}x^5$

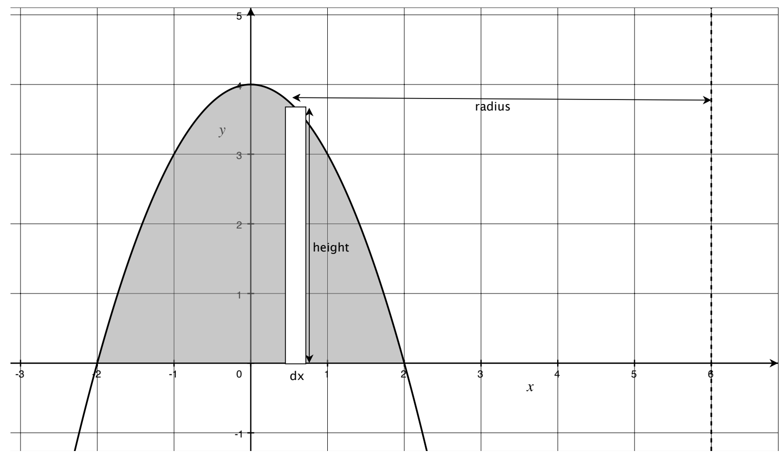
$$\int x^4 \ln(x)\ dx = \frac{1}{5}x^5\ln(x) - \int \frac{1}{5}x^4\ dx = \frac{1}{5}x^5\ln(x) - \frac{1}{25}x^5 + C$$

6. $$v(x) = \frac{x^4 + 4x^3 + x^2 + 7}{x^2(x+2)^3(x^2+4)(x^2-9)} = \frac{x^4 + 4x^3 + x^2 + 7}{x^2(x+2)^3(x^2+4)(x-3)(x+3)}$$
$$= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+2} + \frac{D}{(x+2)^2} + \frac{E}{(x+2)^3} + \frac{F}{x-3} + \frac{G}{x+3} + \frac{Hx+I}{x^2+4}$$

7. The integral is convergent. The integrand is not defined at $x = 1$ so we have
$$\int_{-1}^{2} \frac{1}{\sqrt[3]{x-1}}dx = \lim_{t\to 1^-}\int_{-1}^{t} \frac{1}{\sqrt[3]{x-1}}dx + \lim_{t\to 1^+}\int_{t}^{2} \frac{1}{\sqrt[3]{x-1}}dx$$
$$= \lim_{t\to 1^+} \frac{3}{2}(x-1)^{2/3}\Big]_{-1}^{t} + \lim_{t\to 1^-} \frac{3}{2}(x-1)^{2/3}\Big]_{t}^{2}$$
$$= \lim_{t\to 1^+}\{\frac{3}{2}(t-1)^{2/3} - \frac{3}{2}(-2)^{2/3}\} + \lim_{t\to 1^-}\{\frac{3}{2}(1)^{2/3} - \frac{3}{2}(t-1)^{2/3}\} = -\frac{3}{2}(-2)^{2/3} + \frac{3}{2}$$

8. radius is $6-x$, height is $4-x^2$ so integral is

$$\int_{-2}^{2} 2\pi(6-x)(4-x^2)dx$$



9. (a) diverges

   (b) converges to 1

   (c) $\dfrac{3^n}{(2)^{2n}} = (\dfrac{3}{4})^n$ which converges to 0.

10. (a) The sequence converges to 1/2.

    Let $f(x) = \dfrac{x^3+7}{2x^3+6x^2-x}$. We have

    $$\lim_{x\to\infty} \frac{x^3+7}{2x^3+6x^2-x} = \lim_{x\to\infty} \frac{1+7/x^3}{2+6/x-1/x^2} = \frac{1}{2}$$

    The a_n correspond to values of the function $f(x)$ as $x \to \infty$ and since $f(x) \to 1/2$ as $x \to \infty$, $a_n \to 1/2$ as $n \to \infty$

    (b) The sequence converges to 2.

    We have $0 \leq \dfrac{\cos^2(n)}{n^2} \leq \dfrac{1}{n^2}$ and $\lim\limits_{n\to\infty} 0 = \lim\limits_{n\to\infty} \dfrac{1}{n^2} = 0$. So, by the squeeze theorem, $\lim\limits_{n\to\infty} \dfrac{\cos^2(n)}{n^2} = 0$ and $\lim\limits_{n\to\infty} (2 - \dfrac{\cos^2(n)}{n^2}) = \lim\limits_{n\to\infty} 2 - \lim\limits_{n\to\infty} \dfrac{\cos^2(n)}{n^2} = 2 - 0 = 2$