

No books, notes or calculators are allowed. Cross out what you do not want us to grade. You must show all your work in order to receive full credit. Please write neatly. You are required to sign your exam book. With your signature, you pledge that you have neither given nor received assistance on this exam.

1. (18 points) Evaluate the following integrals.

$$(a) \int \cos^4(5x) dx \quad (b) \int \frac{dx}{x^2(x-1)} \quad (c) \int \frac{\sqrt{x^2-4} dx}{x}$$

2. (12 points) Determine whether each of the following series converges or diverges. Justify your answer. State and check hypotheses of any test, rules or theorems you use.

$$(a) \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \quad (b) \sum_{n=1}^{\infty} \frac{\sqrt{n^2+3}}{15n^2+3n+1}$$

3. (10 points) Find the radius of convergence and interval of convergence for the following power series:

$$\sum_{n=1}^{\infty} \frac{2^n(x-1)^n}{n}$$

4. (10 points) You may use any of the following series to answer this question.

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad R = \infty.$$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \quad R = \infty.$$

$$\cos x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!} \quad R = \infty.$$

$$\tan^{-1} x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)} \quad R = 1.$$

- (a) Find the Maclaurin series for $f(x) = \frac{\tan^{-1}(4x^2)}{x^2}$.
 (b) Evaluate the indefinite integral as a series: $\int x \sin(x^3) dx$.
 (c) Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n 2^n}{3^n n!}$.

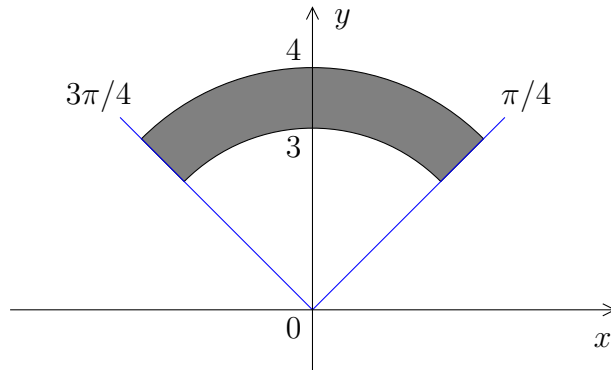
5. (10 points) Let $x(t) = 2 \sin t + 4$, $y(t) = 3 \cos t$, $\pi \leq t \leq 2\pi$.

- (a) Sketch the curve defined by these parametric equations. Indicate with an arrow the direction in which the curve is traced as the parameter increases.
 (b) Write an expression for the length of the curve. **Do not evaluate.**

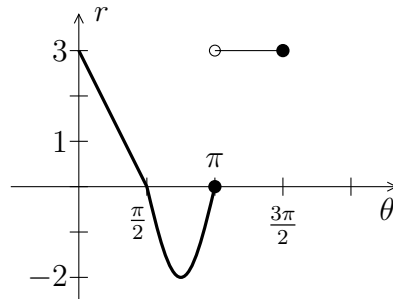
(Exam continues on reverse)

6. (15 points)

- (a) Write down the conditions for r and θ which result in the following shaded region in the plane:



- (b) Find an expression for the area of the region that lies outside the circle $r = 2 \sin \theta$ and inside the circle $r = 1$. **Do not evaluate.**
- (c) The following curve is an (r, θ) Cartesian graph of a polar curve. Sketch it using the polar coordinate system. Label the points where $\theta = 0, \pi/2, \pi,$ and $3\pi/2$.



7. (13 points)

- (a) Find all fourth roots of -16 , write them in the form $a + bi$ and plot them in the complex plane.
- (b) Use De Moivre's theorem to find $(-1 + i\sqrt{3})^6$.
- (c) Write $e^{7+i\pi/6}$ in the form $a + bi$.

8. (12 points)

- (a) Find a simplified form of the Taylor series for $f(x)$ centered at $x = -3$ if

$$f^{(n)}(-3) = \frac{(-1)^n (n+1)!}{7^n \sqrt{n+1}}.$$

- (b) Find the Taylor series for $f(x) = 1/x$ centered at $x = 2$. Write your answer in a simplified form using sigma notation.