Tufts University
Department of Mathematics
Math 12, Exam I

Monday, October 11, 2010   **12:00 p.m.-1:20 p.m.**

No calculators, notes, scratch paper or books are allowed. Except where indicated, you must show all your work in your blue book in order to receive full credit. A correct answer with no work might be given zero points. Cross out any work you do not want graded. Sign your exam book, indicating that you have neither given nor received help during this exam. Any violations will be reported to the appropriate dean, and will result in an F for the course.

1. (6 points): Determine the trigonometric substitution required for each of the following integrals.
   Do NOT evaluate the integrals. No partial credit.

$$(a)\int \sqrt{3x^2 - 1}\, dx \qquad (b)\int \sqrt{x^2 + 6x + 25}\, dx$$

2. (10 points): Write out the form of the partial fraction decomposition of the following rational function and find the constants. **Do NOT integrate anything.** $f(x) = \dfrac{16}{x(x^2 + 4)^2}$

3. (12 points): Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the curves $y = e^{-x}, y = 0, x = 0$, and $x = -1$ about the line $x = 2$.

4. (12 points): Determine if the improper integral

$$\int_{-1}^{1} \frac{2dx}{x^2 - 2x}$$

is convergent or divergent. Evaluate the integral if it is convergent.

5. (10 points): Solve the following initial value problem. Make sure you write y as a function of x.

$$\frac{dy}{dx} = \frac{4 + x}{yx}, \quad x > 1/2, \quad y(1) = -3$$

Exam continues on the back of this page.

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Evaluate the following definite and indefinite integrals.

6. (10 points): $\displaystyle\int \sqrt{9 - x^2}\, dx$

7. (10 points): $\displaystyle\int_0^4 \frac{4x + 5}{x^2 + 16}\, dx$

8. (10 points): $\displaystyle\int \tan^2(2x) \sec^4(2x)\, dx$

9. (10 points): $\displaystyle\int (\sqrt[4]{\sin x})(\cos^3 x)\, dx$

10. (10 points): $\displaystyle\int \ln(x^2 + 1)\, dx$