Math 32 – Exam II – Solutions

1)

(a) $\boxed{F}$: need f to be diff to get that conclusion

(b) $\boxed{F}$: might have no sign change in f' at $x=c$.

(c) $\boxed{T}$

(d) $y' = x^{\sqrt{x}}\left[\dfrac{2+\ln x}{2\sqrt{x}}\right]$ (using log. diff.). $y'=0$ when $2+\ln x=0$
$\ln x=-2$
$\boxed{x=e^{-2}}$

$\boxed{F}$:

(e) $\boxed{T}$: Eqn. of tangent line to $f(x)=\sqrt{x}$ at $x=25$ is $L(x)=\frac{1}{10}(x-25)+5$. So $\sqrt{24.98} \approx L(24.98)=\frac{1}{10}(-0.02)+5$

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(2) (a) $\boxed{x>0}$; (b) $\boxed{\text{None}}$ since $f(0)$ DNE ; (c) $\dfrac{\ln x}{x^2}=0 \to \ln x=0 \to \boxed{x=1}$.

(d) $\lim\limits_{x\to\infty}\dfrac{\ln x}{x^2} \overset{L}{=}_{\frac{\infty}{\infty}} \lim\limits_{x\to\infty}\dfrac{\frac{1}{x}}{2x}=\lim\limits_{x\to\infty}\dfrac{1}{2x^2}=0$ so $\boxed{\text{H.A. at } y=0}$.

(e) $\lim\limits_{x\to 0^+}\dfrac{\ln x}{x^2}=-\infty$ so $\boxed{x=0 \text{ a V.A.}}$.

(f) $\dfrac{1-2\ln x}{x^3}=0 \to 1=2\ln x \to \ln x=\frac{1}{2} \to x=e^{\frac{1}{2}}$

f': 0 ... $+$ ... $e^{1/2}$ ... $-$

$\boxed{\text{INC on } (0,e^{1/2})}$

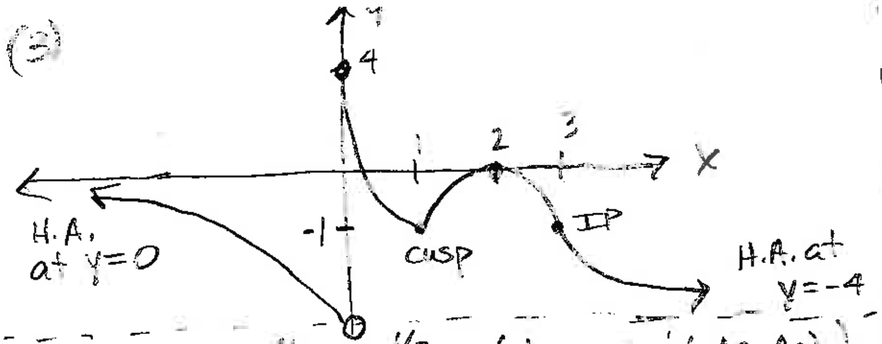
(g) No LOCAL MIN ; (h) $\boxed{\text{local max at } x=e^{1/2}}$

(i) $\dfrac{6\ln x-5}{x^4}=0 \to \ln x=\frac{5}{6} \to x=e^{5/6}$

f'': 0 ... $-$ ... $e^{5/6}$ ... $+$

$\boxed{f \text{ concave up } (e^{5/6},\infty)}$, (j) $\boxed{\text{IP at } x=e^{5/6}}$

(3)



5) $f(x) = 2x^{4/3} - x^{1/3}$ (domain $= (-\infty, \infty)$)

$f'(x) = \frac{8}{3}x^{1/3} - \frac{1}{3}x^{-2/3}$ $\quad f'(0)$ DNE

$f'(x) = \frac{1}{3}x^{-2/3}[8x-1] = 0$ when $x = \frac{1}{8}$

a) $\boxed{\text{Crit Pts: } x = 0, 1/8}$

b) LIST

$f(0) = 0$; $f(1) = 2 - 1 = \boxed{1}$ abs. max

$f(1/8) = 2\left(\frac{1}{8}\right)^{4/3} - \left(\frac{1}{8}\right)^{1/3} = 2\left(\frac{1}{16}\right) - \frac{1}{2}$

$= \boxed{-3/8}$ abs. min

(4) (a) $y = \ln(e^{3x^2} \cdot \tan^{-1}x) \rightarrow$

$y = \ln e^{3x^2} + \ln(\tan^{-1}x) \rightarrow$

$y = 3x^2 + \ln(\tan^{-1}x)$

$y' = 6x + \frac{1}{\tan^{-1}x}\left(\frac{1}{1+x^2}\right)$

$y'(1) = 6 + \frac{1}{\tan^{-1}(1)}\left(\frac{1}{2}\right)$

$= 6 + \frac{1}{\pi/4}\left(\frac{1}{2}\right) = \boxed{6 + \frac{4}{2\pi}}$

(b) $x^3 + y^3 = 2xy$

$3x^2 + 3y^2 \cdot \frac{dy}{dx} = 2x \cdot \frac{dy}{dx} + 2y$

at $(1,1)$ we get:

$3 + 3 \cdot \frac{dy}{dx} = 2 \cdot \frac{dy}{dx} + 2$

$\left.\frac{dy}{dx}\right|_{(1,1)} = -1$

(6) Sps a & b are roots of $f(x)$. Then $f(a) = 0 = f(b)$. Now apply M.V.T. to $f(x)$ on $[a,b]$ to obtain a # c in (a,b) with

$f'(c) = \frac{f(b)-f(a)}{b-a} = \frac{0-0}{b-a} = 0.$

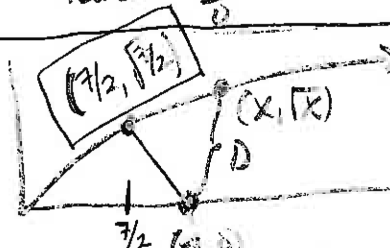
(7) (a) $\lim_{x\to\infty} \frac{x^2+e^x}{x+e^x} \overset{L}{\underset{\infty/\infty}{=}} \lim_{x\to\infty} \frac{2x+e^x}{1+e^x} \overset{L}{\underset{\infty/\infty}{=}} \lim_{x\to\infty} \frac{2+e^x}{e^x} \overset{L}{\underset{\infty/\infty}{=}} \lim_{x\to\infty} \frac{e^x}{e^x} = \boxed{1}$

(b) $\lim_{x\to 0^+} (1+x)^{3\cot x} = e^{D}$ where $D = \lim_{x\to 0^+} \ln(1+x)^{3\cot x}$

$D = \lim_{x\to 0^+} 3\cot x \cdot \ln(1+x) = \lim_{x\to 0^+} \frac{3\ln(1+x)}{\tan x} \overset{L}{\underset{0/0}{=}} \lim_{x\to 0^+} \frac{3 \cdot \frac{1}{1+x}}{\sec^2 x} = \frac{3 \cdot 1}{1} = 3$

so $\boxed{\lim_{x\to 0^+} (1+x)^{3\cot x} = e^3}$

(8)



$D = \sqrt{(4-x)^2 + (\sqrt{x})^2}$

To minimize D, we can minimize

$F(x) = (4-x)^2 + x$ on $[0, \infty)$

$F'(x) = 2(4-x)(-1) + 1$

$F'(x) = 2x - 8 + 1$

$F'(x) = 2x - 7$

$\rightarrow$ $F'(x) = 0$ when $x = \frac{7}{2}$

abs. min for F at $x = \frac{7}{2}$ since 1 sign change in F'