Math 11 Spring 2011 Exam 1 Solutions

1 (a). $\tan(5\pi/3) = \dfrac{\sin(5\pi/3)}{\cos(5\pi/3)} = \dfrac{\sin(-\pi/3)}{\cos(\pi/3)} = \dfrac{-\sqrt{3}/2}{1/2} = \boxed{-\sqrt{3}}$.

(b) since $\sqrt{3} = 3^{1/2}$, $\log_3 \sqrt{3} = \boxed{1/2}$

(c) $= \dfrac{\log_4(12/3)}{\log_4(4^2)} = \dfrac{\log_4(4)}{2} = \boxed{\dfrac{1}{2}}$

2 (a) The eq. $\cos(t) = \dfrac{-1}{\sqrt{2}}$ for t in $[0, 2\pi]$

has solutions $t = \dfrac{3\pi}{4}$ and $t = \dfrac{5\pi}{4}$,

so $\cos(2x) = -1/\sqrt{2}$ for x in $[0, \pi]$ has

solutions

$$2x = \frac{3\pi}{4} \text{ \& } 2x = 5\pi/4$$

i.e. $\boxed{x = 3\pi/8 \text{ and } x = 5\pi/8}$

(b) $\ln(2\ln(x)) = 1 \Rightarrow 2\ln(x) = e^1 = e$

$\Rightarrow \ln(x) = e/2 \Rightarrow \boxed{x = e^{e/2}}$

(c) $e^{3x} = 12 \Rightarrow 3x = \ln(12) \Rightarrow \boxed{x = \frac{1}{3}\ln(12)}$

3 (a) since $(x-1)^2(x-2) > 0$ for $x > 2$, and since $e^x > 0$,

$$\lim_{x \to 2^+} f(x) = +\infty$$

b) since $(x-1)^2(x-2) < 0$ for $1 < x < 2$, and since $e^x > 0$,

$$\lim_{x \to 2^-} f(x) = -\infty$$

c) $\displaystyle\lim_{x \to 1^\pm} f(x) = -\infty \Rightarrow \lim_{x \to 1} f(x) = -\infty$.

p.2

4 a) $\lim_{t\to 0}\frac{2t}{\sin(3t)} = 2\lim_{t\to 0}\frac{3t}{3\sin(3t)} = \frac{2}{3}\underbrace{\lim_{t\to 0}\frac{3t}{\sin(3t)}}_{=1} = \boxed{\frac{2}{3}}$

b) $\boxed{VA: \; x=2}$ since $\lim_{x\to 2^+}\frac{\sqrt{x^2+1}}{x-2} = +\infty$.

$$\lim_{x\to\infty}\frac{\sqrt{x^2+1}}{x-2}\cdot\frac{1/x}{1/x} = \lim_{x\to\infty}\frac{\sqrt{(x^2+1)\cdot 1/x^2}}{1-2/x} = \lim_{x\to\infty}\frac{\sqrt{1+1/x^2}}{1-2/3} = 1$$

since $\sqrt{x^2}=x$ for $x>0$

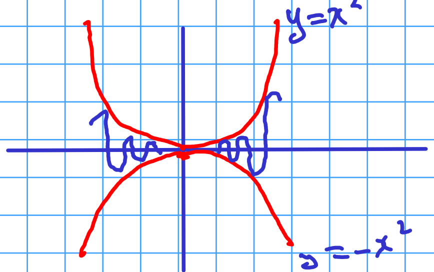
$$\lim_{x\to-\infty}\frac{\sqrt{x^2+1}}{x-2}\cdot\frac{1/x}{1/x} = \lim_{x\to-\infty}\frac{-\sqrt{(x^2+1)\cdot 1/x^2}}{1-2/x} = -1$$

since $\sqrt{x^2}=-x$ for $x<0$

So $\boxed{\text{H Asymptotes: } \; y=1 \; \text{ and } \; y=-1.}$

c) since $-1\le\cos(3/x)\le 1$ for every x, and since $x^2\ge 0$,

conclude that $-x^2\le x^2\cos(3/x)\le x^2$



Since $\lim_{x\to 0}x^2 = \lim_{x\to 0}-x^2 = 0$,

conclude by the $\boxed{\text{Squeeze Theorem}}$

that $\boxed{\lim_{x\to 0}x^2\cos(3/x)=0.}$

5 (1) $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\left(\frac{1}{1+x+h}-\frac{1}{1+x}\right)}{h}\cdot\frac{(1+x)\cdot(1+x+h)}{(1+x)\cdot(1+x+h)}$

$$= \lim_{h\to 0}\frac{(1+x)-(1+x+h)}{h(1+x)(1+x+h)} = \lim_{h\to 0}\frac{-h}{h(1+x)(1+x+h)}$$

$$= \lim_{h\to 0}\frac{-1}{(1+x)(1+x+h)} = \frac{-1}{(1+x)^2}$$

6) a) $\frac{dy}{dx} = \frac{1}{2}\left(x^2 + \frac{1}{x}\right)^{-1/2} \cdot \left(2x - \frac{1}{x^2}\right)$

b) $\frac{dy}{dx} = e^{x^2} \cdot (3x^2) + (x^3+1) \cdot e^{x^2} \cdot (2x)$

c) $\frac{dy}{dx} = \frac{\cos(x)(2x) - (1+x^2)\cdot(-\sin(x))}{\cos^2(x)}$

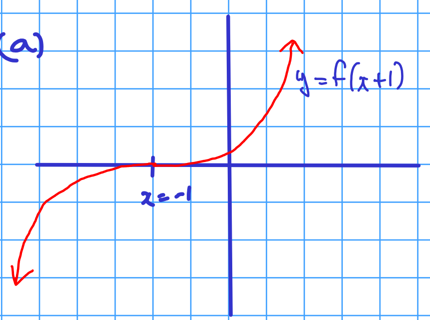
(7) a) Since $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} 2 = 2$ and since $\lim_{x\to 1^+} f(x) = \lim_{x \to 1^+} 3x-1 = 3-1 = 2$

f is continuous at $x=1$

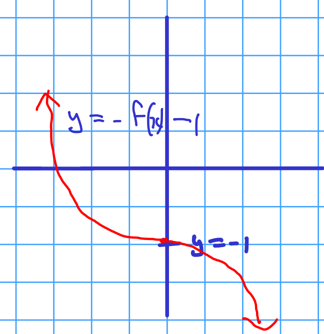
(b) On $(1,\infty)$, $f'(x) = 3$ while on $(-\infty, 1)$ $f'(x) = 0$.

In particular, $\lim_{h \to 0} \frac{f(1+h)-f(1)}{h}$ fails to exist, so f is <u>not</u> diff'able at $x=1$.

(8) (a)



b)



9) a) since $\frac{d}{dx}\tan(2x) = 2\sec^2(2x)$

slope of tang. line at $x = -\pi/8$ is

$$2\sec^2(2x)\Big|_{x=-\pi/8} = \frac{2}{\cos^2(2\cdot(-\pi/8))} = \frac{2}{\cos^2(-\pi/4)} = \frac{2}{(1/\sqrt{2})^2} = 4$$

So tangent line is $y - (-1) = 4(x - (-\pi/8))$

or $y+1 = 4(x + \pi/8)$

b) $\cos(\pi/2) = 0$ so $\tan(2x)$ is not defined at $x = \pi/4$

i.e. $\pi/4$ is <u>not</u> in domain of f.

graph of $y = f(x)$ has a $\boxed{\text{vertical asymptote}}$ at $x = \pi/4$

10) a) $x = \ln(y) + 2 \Rightarrow \ln(y) = x-2 \Rightarrow y = e^{x-2}$

$\Rightarrow \boxed{f^{-1}(x) = e^{x-2}}$

b) Range of f = domain of f^{-1} = $(-\infty, \infty)$

11) Since $f(1) = 2 + 1 - 2 = 1 > 0$

& $f(0) = -2$,

IVT $\Rightarrow$ there is some c in $(0,1)$

s.t. $f(c) = 0$.