Tufts University

Math 11	Department of Mathematics	May 4, 2007
	Final	8:30-10:30

No books, notes, or calculators are allowed on the exam. Please do all of your work in the blue book. Remember to sign your exam book. With your signature, you are pledging that you have neither given nor received help o n this exam.

Part I.

1. (15 points) No partial credit. On the inside front cover of your bluebook, record answers only to (A)-(N) below. None is a possible answer. You may do your scratch work elsewhere in the bluebook.

   Let $f(x) = x + \frac{1}{x}$.

   The domain of $f(x)$ is $\underline{\quad (A) \quad}$.

   The y-coordinate(s) at which $f(x)$ has a horizontal asymptote is (are) $\underline{\quad (B) \quad}$.

   The x-coordinate(s) at which $f(x)$ has a vertical asymptote is (are) $\underline{\quad (C) \quad}$.

   The interval(s) on which $f(x)$ is increasing is (are) $\underline{\quad (D) \quad}$ because $\underline{\quad (E) \quad}$.

   The open interval(s) on which $f(x)$ is concave up is (are) $\underline{\quad (F) \quad}$ because $\underline{\quad (G) \quad}$ or if none explain why.

   The x and y coordinates of each point of inflection of $f(x)$ is (are) $\underline{\quad (H) \quad}$. This (these) is (are) point(s) of inflection because $\underline{\quad (I) \quad}$ or if none explain why.

   The x and y-coordinates at which $f(x)$ has a local maximum are $\underline{\quad (J) \quad}$. because $\underline{\quad (K) \quad}$ or if none explain why.

   The x and y-coordinates at which $f(x)$ has a local minimum are $\underline{\quad (L) \quad}$ because $\underline{\quad (M) \quad}$ or if none explain why.

   (N) Sketch the graph of $f(x)$. Be sure to label any intercepts.

EXAM CONTINUES ON REVERSE SIDE

Part II. You must show your work in Part II to receive full credit.

2. (5 points) Let

$$h(x) = \begin{cases} (x-c)^2 + 1, & \text{if } x \le 1 \\ 2x - 1, & \text{if } x > 1 \end{cases}.$$

   (a) Find the value of c which makes $h(x)$ continuous everywhere.

   (b) Is f differentiable everywhere? Justify your answer.

3. (5 points) Use implicit differentiation to find an equation of the tangent line to the ellipse

$$x^2 + \frac{y^2}{4} = 1 \quad \text{at the point} \quad \left(\frac{\sqrt{2}}{2}, \sqrt{2}\right).$$

4. (10 points; 5 points each) Find the following limits. Be sure to justify the use of L'Hopital's Rule every time it is applied.

   (a) $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$

   (b) $\lim_{n \to \infty} \sum_{i=1}^{n} \frac{4}{n} \sqrt{i \frac{4}{n}}$ [Hint: Re-express the limit by something you can evaluate.]

5. (30 points; 5 points each) [In this part DO NOT SIMPLIFY UNLESS SPECIFIED.]

   Find $g'(x)$ when

   (a) $g(x) = (2x-1)^2(3x+4)^3$ (b) $g(x) = \frac{\sec x}{x^4 + 1}$

   (c) $g(x) = \sin^{-1}[\tan(x \ln x)]$ (d) $g(x) = (\sin x)^{\cos x}$ on $(0, \pi/2)$

   (e) $g(x) = \int_1^{e^{x^2}} \ln y \, dy$ [Simplify your answer as far as possible.]

   (f) Find $g'(x)$ when $g''(x) = x^3 - 5x^2$ and $g'(2) = 4$.

6. (20 points; 5 points each) Evaluate the integrals below. Express answers to (c) and (d) as a number.

   (a) $\int \frac{1+x^2}{x^3} \, dx$ (b) $\int e^x \cos(e^x) \, dx$ (c) $\int_1^{\sqrt{3}} \frac{1}{1+x^2} \, dx$ (d) $\int_0^{\pi/4} \tan^5 x \, \sec^2 x \, dx$

7. (5 points) A rectangle has its base on the x-axis and its upper corners on the graph $y = 3 - x^2$. Find the dimensions of the rectangle which has maximal area. Be sure to justify that your choice gives the maximal area.

8. (5 points) The following integral gives the area of a region: $\int_0^{\sqrt{2}} (\sqrt{4 - x^2} - x) \, dx$. Draw the region and use it to evaluate the integral geometrically.

9. (5 points)

   a) Sketch the region R in the first quadrant bounded by the curves $y = \sin x$, $y = 1$, and $x = 0$.

   b) Using the method of disks or washers, write an integral giving the volume of revolution generated when R is revolved about the y-axis. DO NOT EVALUATE THE INTEGRAL.

END OF EXAM. HAVE A TERRIFIC SUMMER!