Math 11 | **Tufts University**
Department of Mathematics
Exam 2 | **March 30, 2009**

Instructions: No calculators, notes or books are allowed. You should show all work to receive full credit. **Simplify your answers.** Please circle your answers and cross out any work you do not want graded. Remember to sign your blue book, indicating that you have neither given nor received assistance on this exam.

1. (12 pts) Find the derivative. Do not simplify your answer.

   (a) $y = x \tan^{-1} x$ (b) $y = \frac{1}{2}\ln(1+x^2)$ (c) $y = x^{\sin x}$

2. (6 pts) Evaluate the following.

   (a) $\tan^{-1}(1)$. (b) $\cos^{-1}(\cos\left(\frac{13\pi}{6}\right))$.

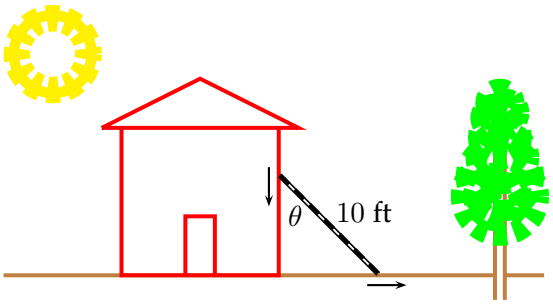
3. (12 pts) Find the limit.

   (a) $\lim_{t\to 1} \frac{\sin(2x-2)}{\sin(3x-3)}$. (b) $\lim_{x\to 0} \frac{e^{2x}-1}{\sin^{-1} x}$. (c) $\lim_{x\to\infty} \left(1-\frac{7}{x}\right)^x$.

4. (12 pts) A ladder 10 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 2 feet per second.

   (a) How fast is the top of the ladder moving down the wall when its base is 6 feet from the wall?

   (b) Find the rate at which the angle between the ladder and the wall of the house is changing when the base of the ladder is 6 feet from the wall.



5. (8 pts) For $xy - 2y^2 = -2$, find the equation of the tangent line to the curve at the point $(3, 2)$.

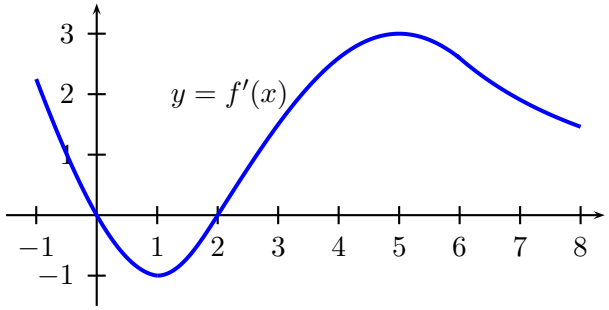
6. (12 pts) For each function below: (i) Find all critical numbers. (ii) Find the function's absolute maximum and minimum on the given closed interval.

   (a) $f(x) = x^3 - 3x - 1$, $[0, 3]$.

   (b) $f(x) = 3x^{\frac{2}{3}} - x$, $[-1, 27]$.

The exam continues on the back.

7. (10 pts) The graph of $f'(x)$ for the interval $[-1, 8]$ is shown below.



   (a) At what x-value(s) does f have a local maximum?

   (b) At what x-value(s) does f have a local minimum?

   (c) On what open interval(s) is f concave up?

   (d) On what open interval(s) is f concave down?

   (e) (Multiple Choice) If $f(4) = 5$, what does the Mean Value Theorem tell us would be the maximum value of $f(6)$? Choose the best answer.
   (i) 5 (ii) 7 (iii) 11 (iv) 18

8. (18 pts) Let $f(x) = \dfrac{-4}{x^2+3}$. Then $f'(x) = \dfrac{8x}{(x^2+3)^2}$ and $f''(x) = \dfrac{24(1-x^2)}{(x^2+3)^3}$.

   Note: A possible answer to any of the following is "there are none".

   (a) What are the equations of any horizontal asymptotes?

   (b) What are the equations of any vertical asymptotes?

   (c) On which open interval(s) is $f(x)$ increasing?

   (d) On which open interval(s) is $f(x)$ decreasing?

   (e) At what x-value(s) does f have a local minimum?

   (f) At what x-value(s) does f have a local maximum?

   (g) On which open interval(s) is $f(x)$ concave up?

   (h) On which open interval(s) is $f(x)$ concave down?

   (i) Find the $x-$coordinate of any inflection point.

9. (4 pts) The conclusion of the Mean Value Theorem is that "there is some c in the interval (a, b) such that $f'(c) = \dfrac{f(b)-f(a)}{b-a}$ ". What are the assumptions on f?

10. (6 pts) Show that $f(x) = 2x^3 + 3x + 1$ has exactly one root by completing the following steps:

    (a) Use the Intermediate Value Theorem to show that f has at least one root.

    (b) Use the Mean Value Theorem or Rolle's Theorem to show that f has exactly one root.