Tufts University

Math 32 | **Department of Mathematics** | **February 21, 2013**

Exam 1 | **12:00 to 1:20 p.m.**

Instructions: No calculators, notes or books are allowed. **You must show all work to receive full credit, unless otherwise indicated.** Please circle your answers and cross out any work you do not want graded. Remember to sign your blue book, indicating that you have neither given nor received assistance on this exam. You are not allowed to use L'Hospital's rule on this exam.

Please put your answers to the first two questions on the inside of the front cover of your blue book.

1. (10pts) True/False or Short-answer Questions. You do not need to justify your answers. There will be no partial credit.

   (a) Suppose $s(t)$ is the position function of an object moving in a straight line at time $t \geq 0$. What does the slope of the line thru $(a, s(a))$ and $(b, s(b))$ on the graph of $y = s(t)$ measure?

   (b) If $h(x) = f(g(x))$ is defined on $(0, 4)$, with $g(2) = 7$ and $g'(2) = 6$, what else is required in order to find $h'(2)$.

   (c) True or False: Suppose $f(2) = 100$, $g(2) = 0$, and $g(x) < 0$ for all $x < 2$. Then $F(x) = \dfrac{f(x)}{g(x)}$ has a vertical asymptote at $x = 2$.

   (d) Does $f(x) = e^{-2x} + 3$ have any horizontal asymptotes? If so, what are they?

   (e) Sketch the graph of a function that is not continuous at $x = 1$, has $f(1) = 4$, and $\lim_{x\to 1} f(x)$ exists.

2. (10 pts) Sketch the graph of a **single** function g that has all of the following properties:

   - g is defined for all real numbers $x \neq 2$.
   - $\lim_{x\to 2^+} g(x) = \infty$, $\lim_{x\to 2^-} g(x) = -\infty$.
   - $\lim_{x\to 4} g(x) = 5$
   - $g(4) = 1$, $g(0) = 0$, $g(-1) = 1$
   - $\lim_{x\to -1} g(x) = 1$, $g'(-1)$ is undefined.
   - $\lim_{x\to -\infty} g(x) = -3$.

3. (12 pts) Let $f(x) = \dfrac{\tan x}{x}$ on $(0, \pi/2)$.

   (a) Find $\lim_{x\to 0+} f(x)$.

   (b) Determine whether $f(x)$ has a vertical asymptote at $x = \pi/2$.

EXAM CONTINUES ON THE OTHER SIDE

4. (12 pts) Consider the function $f(x) = \dfrac{x^3 + 2x^2 - 24x}{2(x^3 - 4x^2)}$.

   (a) Find the domain of $f(x)$.

   (b) Find $\lim_{x \to 4} f(x)$.

   (c) Find all horizontal asymptotes of $f(x)$.

5. (10 pts) Compute the following limits. If a limit tends to $\pm\infty$ say so. If a limit does not exist, explain why. Justify your answers.

   (a) $\lim_{x \to 3+} \dfrac{x}{\sqrt{x-3}}$

   (b) $\lim_{x \to \infty} \dfrac{2 + \sin x}{\sqrt{x}}$ (Hint: Use Squeeze Theorem)

6. (16 pts) In this problem you may use the following fact, if you find it useful: $\lim_{h \to 0} \left(\dfrac{e^h - 1}{h} \right) = 1$.

   (a) Write the limit definition of the derivative $f'(x)$, for a function $f(x)$.

   (b) Use the limit definition of the derivative to compute the derivative, $f'(x)$ when $f(x) = xe^x$.

   (c) Check your answer to part b) by computing the derivative using the rules we have learned.

   (d) What is the slope of the tangent line to curve $y = xe^x$ at the point $x = 0$?

7. (8 pts) Let f be the function defined below. Note: one-sided limits are needed in both parts of this problem.

$$f(x) = \begin{cases} x^2 & x \leq 0 \\ x^3 & x > 0 \end{cases}$$

   (a) Determine whether $f(x)$ is continuous at $x = 0$ by using the definition of continuity at a point.

   (b) Determine whether $f(x)$ is differentiable at $x = 0$ by evaluating the $f'(0)$ using the limit definition.

8. (15pts) Compute the first derivative of each of the following functions using the rules for differentiating. DO NOT SIMPLIFY.

   (a) $f(x) = \dfrac{\tan x}{x}$

   (b) $f(x) = \sec^2(3x^5)$

   (c) $f(x) = e^{\cos x} \cot(\sqrt{x})$

9. (7 pts)

   (a) State the Intermediate Value Theorem.

   (b) Use the Intermediate Value theorem to show there is at least one point in the interval $(0, \pi/4)$ where the function $f(x) = \cos x - 8 \tan x$ takes the value $1/2$.

End of Exam