Tufts University

Math 11 | Department of Mathematics | February 28, 2011

Exam 1

You may not use books, notes, **or calculators** during the exam. Solutions must be written in your exam book; cross out any work you do not want graded. You are required to **sign your exam book**; with your signature, you pledge that you have neither given nor received assistance on this exam.

Don't use l'Hospital's rule to compute limits on this exam.

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1. (12 points) Evaluate the following expressions:

(a) $\tan\left(\frac{5\pi}{3}\right)$ (b) $\log_3(\sqrt{3})$ (c) $\frac{\log_4(12)+\log_4(1/3)}{\log_4(16)}$.

2. (12 points) Find all solutions to the following equations:

(a) $\cos(2x) = \frac{-1}{\sqrt{2}}$ for $0 \le x \le \pi$ (b) $\ln(2\ln(x)) = 1$ (c) $e^{3x} - 12 = 0$

3. (9 points) Consider the function $f(x) = \frac{e^x}{(x-1)^2(x-2)}$. Compute the following limits.

(a) $\lim_{x\to 2^+} f(x)$ (b) $\lim_{x\to 2^-} f(x)$ (c) $\lim_{x\to 1} f(x)$

4. (12 points)

(a) Compute $\lim_{t\to 0} \frac{2t}{\sin(3t)}$.

(b) Find the horizontal and vertical asymptotes of the graph of $y = \frac{\sqrt{1+x^2}}{x-2}$.

(c) Explain why the inequality $-x^2 \le x^2\cos(3/x) \le x^2$ holds for every real number $x \neq 0$. Using this observation, calculate $\lim_{x\to 0} x^2\cos(3/x)$.

5. (8 points)

(a) State the definition of the derivative of a function $f(x)$.

(b) Consider the function $f(x) = \frac{1}{1+x}$. *Use the definition of the derivative* to find $f'(x)$.

6. (12 points) Find the derivative $\frac{dy}{dx}$ of the following functions $y = f(x)$. Do not simplify your answer.

(**a**) $y = \sqrt{x^2 + 1/x}$ (**b**) $y = e^{x^2}(x^3+1)$ (**c**) $y = \frac{1+x^2}{\cos(x)}$

EXAM CONTINUES ON REVERSE.

7. (6 points) Consider the function $f(x) = \begin{cases} 3x - 1 & x \geq 1 \\ 2 & x < 1 \end{cases}$.

(a) Is the function f continuous at $x = 1$? Why or why not?

(b) Is the function f differentiable at $x = 1$? Why or why not?

8. (8 points) Consider the function $y = f(x)$ whose graph is indicated here: In your blue book, sketch the graphs of the following functions:

(a) $y = f(x + 1)$

(b) $y = -f(x) - 1$

9. (8 points) Consider the function $f(x) = \tan(2x)$.

(a) Find the equation of the tangent line to the curve $y = f(x)$ when $x = -\pi/8$.

(b) Is $\pi/4$ in the domain of f? What can be said about the graph of $y = f(x)$ when $x = \pi/4$?

10. (7 points)

(a) The function $f(x) = \ln(x) + 2$ is one-to-one on the domain $(0, \infty)$. Find the inverse function $f^{-1}(x)$.

(b) What is the range of the function $f(x)$? What is the domain of f^{-1}?

11. (6 points) Use the Intermediate Value Theorem to show that for some c in the interval $(0, 1)$, the function

$$f(x) = 2x^3 + x^2 - 2$$

satisfies $f(c) = 0$.

END OF EXAM.