Tufts University

Math 11 Department of Mathematics

Exam 1 February 19, 2009

You may not use books, notes, **or calculators** during the exam. Solutions must be written in your exam book; cross out any work you do not want graded. You are required to **sign your exam book**; with your signature, you pledge that you have neither given nor received assistance on this exam.

Don't use l'Hospital's rule to compute limits on this exam.

1. (9 points) Evaluate the following expressions:

(a) $\log_9(1/3)$ (b) $\sin\left(\frac{-2\pi}{3}\right)$

(c) $\log_{15}(1/2) + \log_{15}(30)$.

2. (9 points) Find all solutions to the following equations:

(a) $\cos(2x) = \frac{\sqrt{3}}{2}$ for $\frac{-\pi}{4} \leq x \leq \frac{\pi}{4}$ (b) $\log_2(\log_2(x) + 2) = 2$

(c) $e^{\ln(x)/2} = 5$

3. (8 points) Consider the function $f(x) = \frac{x^2+1}{(x-3)(x-4)}$. Compute the following limits.

(a) $\lim_{x \to 3^+} f(x)$.

(b) $\lim_{x \to 1} f(x)$.

4. (8 points)

(a) Find the horizontal asymptotes of the graph of $y = \frac{\sqrt{1+2x+9x^2}}{1+2x}$.

(b) Compute $\lim_{x \to 1} \frac{\sin(2x-2)}{\sin(3x-3)}$.

5. (12 points) Find the derivative $\frac{dy}{dx}$ of the following functions $y = f(x)$. Do not simplify your answer.

(**a**) $y = x \cdot e^{2x}$ (**b**) $y = \sin(\sqrt{x+1})$ (**c**) $y = \frac{1+x^3}{\cos(x)}$

6. (10 points)

(a) State the definition of the derivative of a function f.

(b) Consider the function $f(x) = \sqrt{x}$. *Using the definition*, find the formula for the derivative $f'(x)$.

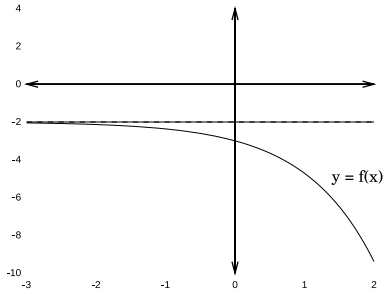
7. (8 points) Let $F(x)$ be a function for which $F(1) = 3, F(2) = 1, F'(1) = 1$. For each of the following functions $y = g(x)$, find the value $\frac{dy}{dx}|_{x=1} = g'(1)$ of the derivative of y when $x = 1$:

(a) $y = F(x^2)$,

(b) $y = x^2 F(x)$.

EXAM CONTINUES ON REVERSE.

8. (4 points) The following is the graph $y = f(x)$. Choose the formula for $f(x)$ which best corresponds to the graph. *Indicate your choice in your blue book; no partial credit will be awarded.*



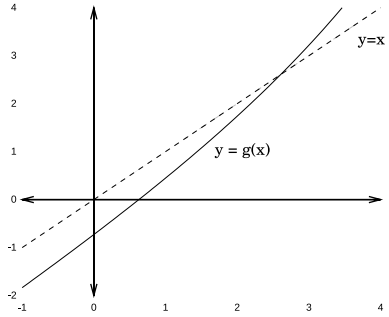
(**a**) $f(x) = e^{-(x+2)}$ (**b**) $f(x) = -e^{(x+2)}$
(**c**) $f(x) = -e^{x} - 2$ (**d**) $f(x) = -e^{-x} - 2$

9. (8 points)

(a) The function $f(x) = e^x + 2$ is one-to-one. Find the inverse function $f^{-1}(x)$.

(b) What is the range of the function $f(x)$? what is the domain of f^{-1}?

(d) Consider the following diagram. The solid curve is the graph of a function $y = g(x)$, and the dashed line is the graph of $y = x$. The function $g(x)$ is one-to-one. Indicate whether the following statements are *True or False* in you bluebook. *No partial credit will be awarded.*



(i) $g^{-1}(1) < 0$.

(iii) $g^{-1}(x) = 0$ for some x in the interval $[-1, 1]$.

(iv) $g^{-1}(x) = x$ for some x in the interval $[2, 3]$.

10. (6 points)

(a) Sketch the graph of a function f with the following properties:

$$f(1) = f'(1) = 0, \quad \lim_{x \to 5^+} f(x) = \infty, \quad \lim_{x \to 5^-} f(x) = -\infty, \quad \lim_{x \to \infty} f(x) = 1, \quad \lim_{x \to -\infty} f(x) = -1$$

(b) Give the equations for all asymptotes of your function f.

11. (8 points) Consider the function $f(x) = \begin{cases} 3x + 1 & x \geq 1 \\ x^2 + x + 2 & x < 1 \end{cases}$.

(a) Is the function f continuous at $x = 1$? why or why not?

(b) Is the function f differentiable at $x = 1$? why or why not?

12. (10 points) Find the equation of the tangent line to the curve $y = \dfrac{1}{(1+x)^2}$ which is parallel to the line $4y + x = 1$.

END OF EXAM.