Math 32 | Tufts University
Department of Mathematics
Final Exam Review | Spring 2014

1. Find the function $f(x)$ determined by the given conditions.
   (a) $f''(x) = 20x^3 + 12x^2 + 4, \qquad f(0) = 8, f(1) = 5$
   (b) $f'''(x) = \sin x, \qquad f(0) = 1, f'(0) = 1, f''(0) = 1$

2. Evaluate the following integrals. Note: some may need to be found geometrically.

   (a) $\displaystyle\int_1^4 \frac{1}{\sqrt{x}}dx$

   (b) $\displaystyle\int_{-\frac{\pi}{3}}^{\frac{\pi}{2}} \sin(2x)dx$

   (c) $\displaystyle\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{6}{\sqrt{1-x^2}}dx$

   (d) $\displaystyle\int_0^4 \sqrt{16-x^2}dx$

   (e) $\displaystyle\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \tan^2 x \sec^2 x \, dx$

   (f) $\displaystyle\int_1^2 \frac{4+u^2}{u^3}du$

   (g) $\displaystyle\int_0^4 \frac{1}{(x-2)^3}dx$

   (h) $\displaystyle\int_e^{e^4} \frac{1}{x\sqrt{\ln x}}dx$

3. Evaluate the following indefinite integrals.

   (a) $\displaystyle\int \frac{x}{\sqrt{1+x}}dx$ (b) $\displaystyle\int \frac{\sec^2\sqrt{x}}{\sqrt{x}}dx$ (c) $\displaystyle\int \frac{\sin x}{1+\cos^2 x}dx$

4. Find the derivative. Do not simplify.

   (a) $g(u) = \displaystyle\int_3^u \frac{1}{x+x^2}\,dx$

   (b) $f(x) = \displaystyle\int_1^x \ln t dt$

   (c) $f(x) = \displaystyle\int_3^{\sqrt{x}} \frac{\cos t}{t}\,dt$

   (d) $f(x) = \displaystyle\int_1^{\cos x} (t+\sin t)\,dt$

   (e) $f(x) = e^{x^2} + 1$

   (f) $f(x) = \dfrac{\ln x}{\sin x}$

   (g) $y = x^x$

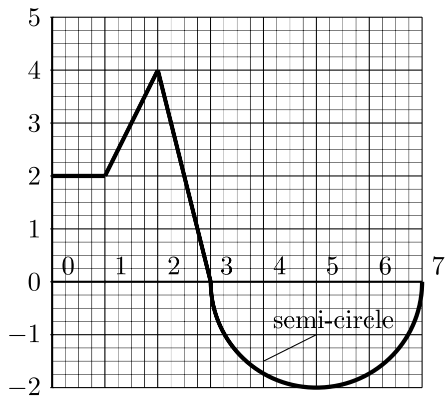
   (h) $y = \dfrac{2^x\sqrt{4+\cos x}}{(x^2+x+2)^5}$

5. Find the area of the regions bounded by the following curves.
   (a) $4x + y^2 = 12$ and $y = x$
   (b) $y = x^2 - 2$ and $y = x$
   (c) $y = e^x, y = e^{-x}$, and $x = -2$

6. A cylindrical container is said to have a volume of 10m^3. The bottom and sides of the container are made of sheet metal, while the top is **open**. Find the dimensions of the container which requires the smallest amount of metal.

7. What is the largest possible volume of a box that can be made from a 5×8 in rectangular piece of cardboard, by cutting out squares at the corners and folding up the edges? (The box has bottom and sides but is open on top).

8. The graph of $f(x)$ is given below. Use it to answer the following questions.



(a) Evaluate $\int_1^5 f(x)dx$

(b) Where does $g(x) = \int_0^x f(t)dt$ have a minimum value?

(c) In which open interval is $g(x) = \int_0^x f(t)dt$ concave down?

9. Find the equation of the tangent line to the curve $x^4 + y^4 - 4xy = 9$ at the point $(2, 1)$.

10.

(a) State the Intermediate Value Theorem.

(b Give an example of a function that does **not** satisfy the conclusion of the Intermediate Value Theorem on the interval [0,1].

(c) State the Mean Value Theorem.

(d) Given an example of a continuous function that does **not** satisfy the conclusion of the Mean Value Theorem on the interval [0,1].

11.

(a) State the limit definition of the derivative.

(b) Use the limit definition of the derivative to find $f'(x)$ where $f(x) = \dfrac{1}{3x-1}$.

12. Find each limit. If you use L'Hospital's rule, indicate which kind of indeterminate form is involved. Remember: You can only apply L'Hospital's rule directly if you have type 0/0 or ∞/∞!

(a) $\lim\limits_{x\to 0+} \left(\dfrac{1}{x} - \dfrac{1}{\sin x}\right)$ (b) $\lim\limits_{x\to\infty} \dfrac{\ln(\ln x)}{x}$ (c) $\lim\limits_{x\to\infty} (e^x + 1)^{\frac{1}{x}}$

(d) $\lim\limits_{x\to 2-} \dfrac{x^2 + x - 2}{x^2 - 4}$ (e) $\lim\limits_{x\to\infty} (5x - \sqrt{x^2 + 3x})$ (f) $\lim\limits_{x\to 2-} \dfrac{x^2 - x + 2}{x^2 - 4}$

13. Given

$$f(x) = \frac{x}{(x-1)^2} \qquad f'(x) = -\frac{(x+1)}{(x-1)^3} \qquad f''(x) = \frac{2x+4}{(x-1)^4}$$

Answer the following questions about the graph $y = f(x)$

(a) Find the $x-$ and $y-$intercepts.

(b) Find any horizontal and/or vertical asymptotes. Compute the approximate limits to justify your answers.

(c) Find the intervals where f is increasing and where f is decreasing.

(d) Find the $x-$ and $y-$ coordinates of any local minimums or local maximums.

(e) Find the intervals where f is concave up and where f is concave down.

(f) Find the $x-$ and $y-$coordinates of any inflection points.

(g) Sketch the graph of $y = f(x)$, indicating the features you have found (i.e. label the $x-$ and $y-$intercepts, any asymptotes, local min/max points, and inflection points).