1.

(a) $f''(x) = 20x^3 + 12x^2 + 4 \qquad f'(x) = \frac{20x^4}{4} + \frac{12x^3}{3} + 4x + C = 5x^4 + 4x^3 + 4x + C$

$f(x) = \frac{5x^5}{5} + \frac{4x^4}{4} + \frac{4x^2}{2} + Cx + D = x^5 + x^4 + 2x^2 + Cx + D$

$f(0) = D$, so $D = 8$. $\qquad f(1) = 4 + C + 8$, so $5 = 12 + C$ and therefore $C = -7$

$\boxed{f(x) = x^5 + x^4 + 2x^2 - 7x + 8}$

(b) $f'''(x) = \sin x \qquad f''(x) = -\cos x + C$, so $f''(0) = -\cos 0 + C = 1 + C$

$1 = -1 + C \qquad C = 2, \qquad$ so $f''(x) = -\cos x + 2$

$f'(x) = -\sin x + 2x + D \qquad f'(0) = 0 + 2(0) + D, \qquad 1 = D, \qquad f'(x) = -\sin x + 2x + 1$

$f(x) = \cos x + x^2 + x + E \qquad f(0) = 1 + 0 + 0 + E \qquad 1 = 1 + E \qquad E = 0$

$\boxed{f(x) = \cos x + x^2 + x}$

2.

(a) $\int_1^4 \frac{1}{\sqrt{x}} dx = \int_4^1 x^{-\frac{1}{2}} dx = 2x^{\frac{1}{2}}\Big]_1^4 = 2\sqrt{4} - 2\sqrt{1} = 4 - 2 = \boxed{2}$

(b) $\int_{-\frac{\pi}{3}}^{\frac{\pi}{2}} \sin(2x)dx = \frac{-cos(2x)}{2}\Big]_{-\frac{\pi}{3}}^{\frac{\pi}{2}} = \frac{-\cos(2\cdot\frac{\pi}{2})}{2} - \left(\frac{-\cos(2\cdot\frac{-\pi}{3}))}{2}\right) = \frac{1}{2} + \frac{\cos\left(\frac{-2\pi}{3}\right)}{2}$

$= \frac{1}{2} + \frac{-\frac{1}{2}}{2} = \frac{1}{2} - \frac{1}{4} = \boxed{\frac{1}{4}}$

(c) $\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} \frac{6}{\sqrt{1-x^2}} dx = 6\sin^{-1} x\Big]_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}} = 6\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - 6\sin^{-1}\left(\frac{1}{2}\right) = 6\cdot\frac{\pi}{3} - 6\cdot\frac{\pi}{6} = 2\pi - \pi = \boxed{\pi}$

(d) $\int_0^4 \sqrt{16 - x^2}dx$ is the area the circle of radius 4 in the first quadrant, hence $= \frac{\pi(4)^2}{4} = \boxed{4\pi}$.

(e) $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \tan^2 x \sec^2 x dx \qquad [u = \tan x,\ du = \sec^2 x dx$; if $x = \frac{\pi}{3}, u = \tan\frac{\pi}{3} = \sqrt{3}$;

if $x = -\frac{-\pi}{3},\ u = \tan -\frac{\pi}{3} = \frac{-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = -\sqrt{3}]$

$= \int_{-\sqrt{3}}^{\sqrt{3}} u^2 du = \frac{u^3}{3}\Big]_{-\sqrt{3}}^{\sqrt{3}} = \frac{(\sqrt{3})^3}{3} - \frac{(-\sqrt{3})^3}{3} = \frac{\sqrt{27}+\sqrt{27}}{3} = \frac{2\cdot 3\sqrt{3}}{3} = \boxed{2\sqrt{3}}$

(f) $\int_1^2 \frac{4+u^2}{u^3} du = \int_1^2 4u^{-3} + \frac{1}{u} du = = \frac{4u^{-2}}{-2} + \ln|u|\Big]_1^2 = \frac{-2}{u^2} + \ln|u|\Big]_1^2 = \frac{-2}{4} + \ln 2 - \left(\frac{-2}{1} + \ln 1\right)$

$= \frac{-1}{2} + \ln 2 + 2 - 0 = \boxed{\frac{3}{2} + \ln 2}$

(g) $\int_0^4 \frac{1}{(x-2)^3} dx \qquad f(x) = \frac{1}{(x-2)^3}$ is not continuous on [0,4] because it has an infinite discontinuity at $x = 2$ so the integral does not exist.

(h) $\displaystyle\int_e^{e^4} \frac{1}{x\sqrt{\ln x}}dx = \int_e^{e^4} \frac{1}{\sqrt{u}}du$ $\quad [u = \ln x, du = \frac{1}{x}dx;$ if $x = e, u = \ln e = 1;$ if $x = e^4, u = \ln e^4 = 4]$

$\displaystyle = \int_e^{e^4} u^{-\frac{1}{2}} du = 2u^{\frac{1}{2}}\Big]_1^4 = 4 - 2 = \boxed{2}$

3.

(a) $\displaystyle\int \frac{x}{\sqrt{1+x}}dx = \int \frac{u-1}{\sqrt{u}}du$ $\quad$ [where $u = 1 + x$, $x = u - 1$, $du = dx$]

$\displaystyle = \int u^{\frac{1}{2}} - u^{-\frac{1}{2}} = \frac{2}{3}u^{\frac{3}{2}} - u^{\frac{1}{2}} + C = \boxed{\frac{2}{3}(1+x)^{\frac{3}{2}} - 2(1+x)^{\frac{1}{2}} + C}$

(b) $\displaystyle\int \frac{\sec^2\sqrt{x}}{\sqrt{x}}dx = 2\int \sec^2 u \; du$ $\quad$ [where $u = \sqrt{x}$, $du = \frac{1}{2}x^{-\frac{1}{2}}dx$, $2du = \frac{1}{\sqrt{x}}dx$]

$= 2\tan u + C = \boxed{2\tan\sqrt{x} + C}$

(c) $\displaystyle\int \frac{\sin x}{1+\cos^2 x}dx = -\int \frac{1}{1+u^2}dx$ $\quad$ [where $u = \cos x$, $du = -\sin x \; dx$, $-du = \sin x \; dx$]

$= -\tan^{-1} u + C = \boxed{-\tan^{-1}(\cos x) + C}$

4.

(a) $\displaystyle g(u) = \int_3^u \frac{1}{x+x^2}dx$ $\quad \boxed{g'(u) = \frac{1}{u+u^2}}$ by FTC1

(b) $\displaystyle f(x) = \int_1^x \ln t \; dt$ $\quad \boxed{f'(x) = \ln x}$ by FTC1

(c) $\displaystyle f(x) = \int_3^{\sqrt{x}} \frac{\cos t}{t}dt$ $\quad \displaystyle f'(x) = \frac{\cos\sqrt{x}}{\sqrt{x}} \cdot \frac{1}{2}x^{-\frac{1}{2}} = \boxed{\frac{\cos\sqrt{x}}{2x}}$ by FTC1 and the chain rule

(d) $\displaystyle f(x) = \int_1^{\cos x} (t + \sin t)dt$ $\quad \boxed{f'(x) = [\cos x + \sin(\cos x)] \cdot (-\sin x)}$ by FTC1 and the chain rule

(e) $f(x) = e^{x^2+1}$ $\quad \boxed{f'(x) = 2xe^{x^2+1}}$

(f) $\displaystyle f(x) = \frac{\ln x}{\sin x}$ $\quad \boxed{f'(x) = \frac{(\sin x) \cdot \frac{1}{x} - (\ln x)(\cos x)}{\sin^2 x}}$

(g) $y = x^x$ Use logarithmic differentiation: $\ln y = x \ln x$ $\quad \displaystyle\frac{y'}{y} = x \cdot \frac{1}{x} + (\ln x) \cdot 1$ $\quad y' = (1 + \ln x)y$

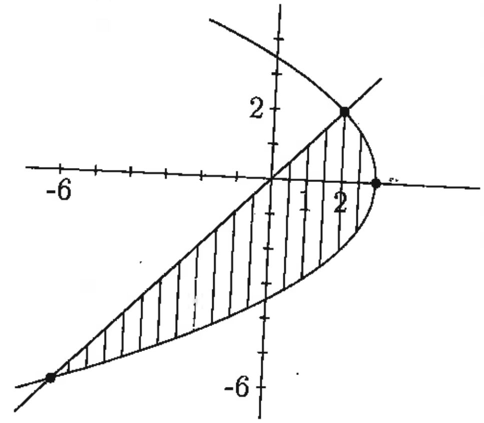
$\boxed{y' = (1 + \ln x)x^x}$

(h) $\displaystyle y = \frac{2^x \cdot \sqrt{4+\cos x}}{(x^2+x+2)^5}$ Use logarithmic differentiation: $\displaystyle\ln y = x\ln 2 + \frac{1}{2}\ln(4+\cos x) - 5\ln(x^2+x+2)$

$\displaystyle\frac{y'}{y} = \ln 2 + \frac{1}{2} \cdot \frac{-\sin x}{4+\cos x} - 5 \cdot \frac{(2x+1)}{x^2+x+2}$ $\quad \boxed{y' = \left[\ln 2 - \frac{\sin x}{2(4+\cos x)} - \frac{5(2x+1)}{x^2+x+2}\right]\left[\frac{2^x\sqrt{4+\cos x}}{(x^2+x+2)^5}\right]}$

5.

(a) Find the area of the region bounded by $4x + y^2 = 12$ and $y = x$.



$x = \dfrac{12 - y^2}{4} = 3 - \dfrac{1}{4}y^2$ so $4x + x^2 = 12$

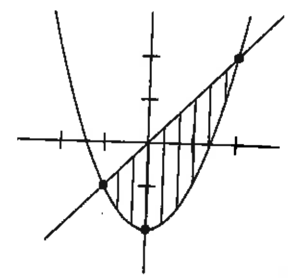
$0 = x^2 + 4x - 12 = (x+6)(x-2)$

$x = -6, 2$

$$\int_{-6}^{2} \frac{12 - y^2}{4} - y\, dy = \int_{-6}^{2} 3 - \frac{y^2}{4} - y\, dy = 3y - \frac{y^3}{12} - \frac{y^2}{2}\Big]_{-6}^{2}$$

$$= \left(6 - \frac{8}{12} - 2\right) - \left(-18 - \frac{6^3}{12} - \frac{36}{2}\right) = 22 - \frac{2}{3} = \boxed{21\ \frac{1}{3}}$$

(b) $y = x^2 - 2$ and $y = x$ so $x^2 - 2 = x$



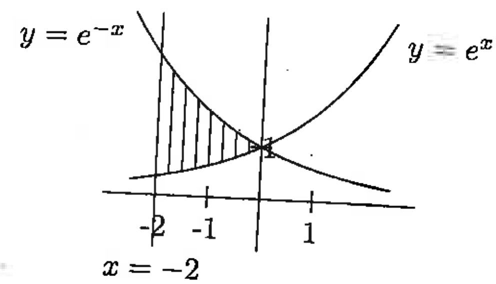
$0 = x^2 - x - 2 = (x-2)(x+1)$

$x = 2, -1$

$$\int_{-1}^{2} x - (x^2 - 2)\, dx = \int_{-1}^{2} x - x^2 + 2\, dx = \frac{x^2}{2} - \frac{x^3}{3} + 2x\Big]_{-1}^{2}$$

$$= \left(2 - \frac{8}{3} + 4\right) - \left(\frac{1}{2} + \frac{1}{3} - 2\right) = \boxed{4\ \frac{1}{2}}$$

(c) $y = e^x, y = e^{-x}$, and $x = -2$



$e^x = e^{-x} \quad e^x = \frac{1}{e^x} \quad e^{2x} = 1$

Thus, $2x = \ln 1$, so $x = 0$

$$\int_{-2}^{0} e^{-x} - e^x\, dx = -e^{-x} - e^x\Big]_{-2}^{0} = -e^0 - e^0 - \left(-e^{-(-2)} - e^{-2}\right)$$

$$= -1 - 1 - (e^2 - e^{-2}) = \boxed{-2 + e^2 + \frac{1}{e^2}}$$

8.

(a) $\displaystyle\int_5^1 f(x)dx = (1)(2) + \frac{1}{2}(1)(2) + \frac{1}{2}(1)(4) - \frac{\pi(2)^2}{4} = 2 + 1 + 2 - \pi = \boxed{5 - \pi}$

(b) $g'(x) = f(x) \qquad g'(x) = 0$ when $x = 3 \qquad$ No local min

Does it have an abs. min? Yes, at $\boxed{x = 0}$ since $g(0) = \int_0^0 f(t)dt = 0$ min and $g(7) = \int_0^7 f(t)dt = 2 + 5 - \pi - \pi = 7 - 2\pi > 0$

(c) $g'(x) = f(x) \qquad g''(x) = f'(x)$

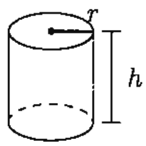
0 DNE+DNE - 0 DNE

0 1 2 5 7

$g''(x) = f'(x)$

CD on $(2, 5)$

6.



Given: $V = 10\text{m}^3$

Goal: minimize surface area

$S = 2\pi r h + \pi r^2$

$S(r) = 2\pi r\left(\frac{10}{\pi r^2}\right) + \pi r^2 = \frac{20}{r} + \pi r^2$

Domain: $r > 0$

$V = \pi r^2 h \quad 10 = \pi r^2 h \quad h = \frac{10}{\pi r^2}$

$S'(r) = -20r^{-2} + 2\pi r$

$\underline{S'(r) = 0}: \quad 2\pi r = \frac{20}{r^2} \quad \pi r^3 = 20$

$r = \left(\frac{10}{\pi}\right)^{\frac{1}{3}}$

$S''(r) = 40r^{-3} + 2\pi = \frac{40}{r^3} + 2\pi$

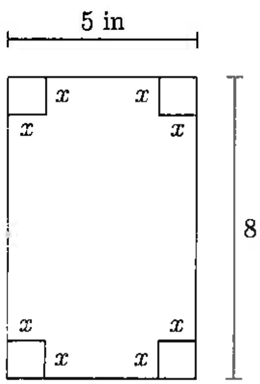
> 0 for all r in the domain

	+	$S''(r)$
	CU	$S(r)$

Since $S'((\frac{10}{\pi})^{\frac{1}{3}}) = 0$ and $S''(r) > 0$ for all r, then we have an absolute min at $r = (\frac{10}{\pi})^{\frac{1}{3}}$.

Dimensions: $\boxed{r = \left(\frac{10}{\pi}\right)^{\frac{1}{3}}, h = \frac{10}{\pi(\frac{10}{\pi})^{\frac{2}{3}}} = \frac{10 \cdot \pi^{\frac{2}{3}}}{\pi 10^{\frac{2}{3}}} = \frac{10^{\frac{1}{3}}}{\pi^{\frac{1}{3}}} = \left(\frac{10}{\pi}\right)^{\frac{2}{3}}}$

7.



Goal: maximize volume

$V = (8 - 2x)(5 - 2x)x$

Domain:

$8 - 2x \geq 0 \quad 8 \geq 2x \quad x \leq 4$

$5 - 2x \geq 0 \quad 5 \geq 2x \quad x \leq \frac{5}{2}$

$x \geq 0$

$[0, \frac{5}{2}]$

$V = (40 - 16x - 10x + 4x^2)x = 40x - 26x^2 + 4x^3 \quad V'(x) = 40 - 52x + 12x^2 = 4(3x^2 - 13x - 10) = 0$

if $3x^2 - 13x + 10 = 0$, i.e. $x = \frac{-(-13) \pm \sqrt{169 - 4(3)(10)}}{6} = \frac{13 \pm \sqrt{49}}{6} = \frac{13 \pm 7}{6}$ so $x = \frac{20}{6}, 1$

20/6 is too big, not in the domain.

Closed interval method: V(5/2)=0, V(0)=0, V(1)= 18 (max) $\boxed{\text{Largest possible volume is } 18\text{in}^3}$

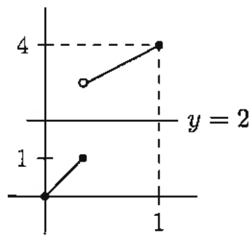
9. $\frac{d}{dx}(x^4 + y^4 - 4xy = 9)$: $\quad 4x^3 + 4y^3\frac{dy}{dx} - \left(4x\frac{dy}{dx} + 4y\right) = 0$

$\frac{dy}{dx}(4y^3 - 4x) = 4y - 4x^3 \qquad \frac{dy}{dx} = \frac{4y - 4x^3}{4y^3 - 4x}$

$m = \frac{dy}{dx}\Big|_{(2,1)} = \frac{4(1) - 4(2)^3}{4(1)^3 - 4(2)} = \frac{-28}{-4} = 7 \qquad \boxed{y - 1 = 7(x - 2)}$

10. (a) Intermediate Value Thm.: If $f(x)$ is continuous on $[a,b]$ and $f(a) < N < f(b)$ (or $f(b) < N < f(a)$), then there is a c in (a,b) so that $f(c) = N$.

(b)



f never crosses the line $y = 2$.

f a function on [0,1]

$f(0) = 0 \quad f(1) = 4 \quad N = 2$

$0 < 2 < 4$

But f is not continuous on [0,1].

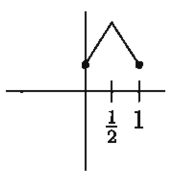
There is NO c in (0,1) so that $f(c) = 2$.

(c) If $f(x)$ is continuous on $[a,b]$ and differentiable on (a,b) then there is a c in (a,b) so that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

(at some point between a and b the slope of the tangent line to the curve is equal to the slope of the secant line).

(d)



The slope of the secant line between 0 and 1 is 0, but there is no c so that $f'(c) = 0$.

$f'(\frac{1}{2})$ DNE. f is not differentiable on (0,1).

11.

(a) $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

(b) $f'(x) = \lim_{h \to 0} \left(\dfrac{1}{3(x+h) - 1} - \dfrac{1}{3x - 1} \right) \cdot \dfrac{1}{h}$

$$= \lim_{h \to 0} \frac{3x - 1 - (3(x+h) - 1)}{h(3(x+h) - 1)(3x - 1)} = \lim_{h \to 0} \frac{3x - 1 - 3x - 3h + 1}{h(3(x+h) - 1)(3x - 1)} = \lim_{h \to 0} \frac{-3h}{h(3(x+h) - 1)(3x - 1)}$$

$$= \lim_{h \to 0} \frac{-3}{(3(x+h) - 1)(3x - 1)} = \boxed{\frac{-3}{(3x - 1)^2}}$$

12.

(a) $$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right) =^{\infty - \infty} \lim_{x \to 0} \frac{\sin x - x}{x \sin x} =^{0/0} \lim_{x \to 0} \frac{\cos x - 1}{x \cos x + \sin x} =^{0/0} \lim_{x \to 0} \frac{-\sin x}{-x \sin x + \cos x + \cos x}$$
$$= \frac{0}{2} = \boxed{0}$$

(b) $$\lim_{x \to \infty} \frac{\ln(\ln x)}{x} =^{\infty/\infty} \lim_{x \to \infty} \frac{\frac{\frac{1}{x}}{\ln x}}{1} = \lim_{x \to \infty} \frac{1}{x \ln x} = \text{“}\frac{1}{\infty}\text{”} = \boxed{0}$$

(c) $\lim_{x\to\infty} (e^x+1)^{\frac{1}{x}} =^{\infty^0} \lim_{x\to\infty} e^{\ln\left((e^x+1)^{\frac{1}{x}}\right)} = e^{\lim_{x\to\infty}\frac{1}{x}\ln(e^x+1)} = \boxed{e^1}$ since:

$$\lim_{x\to\infty} \frac{\ln(e^x+1)}{x} = \lim_{x\to\infty} \frac{\frac{e^x}{e^x+1}}{1} = \lim_{x\to\infty} \frac{e^x}{e^x+1} =^{\infty/\infty} \lim_{x\to\infty} \frac{e^x}{e^x} = 1$$

(d) $\lim_{x\to 2-} \frac{x^2+x-2}{x^2-4} = \lim_{x\to 2-} \frac{(x+2)(x-1)}{(x+2)(x-2)} = \lim_{x\to 2-} \frac{x-1}{x-2} = \boxed{-\infty}$

(e) $\lim_{x\to\infty} 5x - \sqrt{x^2+3x} =^{\infty-\infty} \lim_{x\to\infty} \frac{(5x-\sqrt{x^2+3x})(5x+\sqrt{x^2+3x})}{(5x+\sqrt{x^2+3x})}$

$$= \lim_{x\to\infty} \frac{25x^2-(x^2+3x)}{5x+\sqrt{x^2+3x}} = \lim_{x\to\infty} \frac{24x^2-3x}{5x+\sqrt{x^2+3x}} = \lim_{x\to\infty} \frac{\frac{24x^2}{x}-\frac{3x}{x}}{\frac{5x}{x}+\sqrt{\frac{x^2}{x^2}+\frac{3x}{x^2}}} = \lim_{x\to\infty} \frac{24x-3}{5+\sqrt{1+\frac{3}{x}}} = \boxed{\infty}$$

(f) $\lim_{x\to 2-} \frac{x^2-x+2}{x^2-4} = \lim_{x\to 2-} \frac{(x-2)(x+1)}{(x-2)(x+2)} = \lim_{x\to 2-} \frac{x+1}{x+2} = \boxed{\frac{3}{4}}$

13.

(a) $f(0) = \frac{0}{1} = 0$ $\quad f(x) = 0$ where $x = 0$ $\quad \boxed{(0,0)}$ is the x and y intercept.

(b) $\lim_{x\to\infty} \frac{x}{(x-1)^2} = \lim_{x\to\infty} \frac{x}{x^2-2x+1} = \lim_{x\to\infty} \frac{\frac{1}{x}}{1-\frac{2}{x}+\frac{1}{x^2}} = 0$

Similarly, $\lim_{x\to-\infty} \frac{x}{(x-1)^2} = 0$ $\quad$ So $\boxed{y = 0 \text{ is the only horizontal asymptote}}$

Domain of $f(x)$ is $x \neq 1$ $\ (-\infty,1)\bigcup(1,\infty)$

Does $f(x)$ have a vertical asymptote at $x = 1$?

$\lim_{x\to 1-} \frac{x}{(x-1)^2} = \infty\ ](x<1)$ $\quad \lim_{x\to 1+} \frac{x}{(x-1)^2} = \infty$ $\quad$ Yes, $\boxed{\text{V.A. at x} = 1}$

(c) $\underline{f'(x) = 0:}$ at $x = -1$ $\quad f'(-2) = \frac{-(-1)}{-27} = \frac{-1}{27}$ $\quad f'(0) = 1$ $\quad f'(2) = -3$

- 0 + DNE - $\quad f'(x)$

-1 $\quad$ 1

↘ ↗ ↘ $\quad f(x)$

$\boxed{f \text{ inc. on } (-1,1),\ \text{dec. on } (-\infty,-1),(1,\infty)}$

(d) $f(x)$ has a local min at $x = -1$, no local max. $\quad$ Local min at $(-1, f(-1)) = (-1, -\frac{1}{4})$

(e) $\underline{f''(x) = 0:}$ $\quad \frac{2x+4}{(x-1)^4} = 0$ when $x = -2$

- 0 +DNE- $\quad f''(x)$

-2 $\quad$ 1 $\quad f'(x)$

CD $\quad$ CU $\quad$ CD

$\boxed{f \text{ concave down on } (-\infty,-2),\ \text{concave up } (-2,1),(1,\infty)}$

(f) $f(x)$ has an inflection point at $(-2, f(-2)) = (-2, \frac{-2}{9})$.

Sketch of graph on the next page.

(g)