Math 32 KEY Exam 1 2014

1. a.) $2\ln(6) = \ln(6^2) = \ln(36)$, so $\frac{2\ln(6) - \ln(36)}{\ln(7)} = 0$.

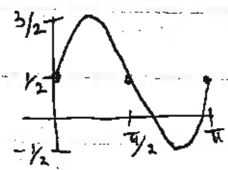
b.) $\sec(11\pi/6) = 1/\cos(11\pi/6)$, $\cos(11\pi/6) = \frac{\sqrt{3}}{2}$, so $1/\frac{\sqrt{3}}{2} = 2/\sqrt{3}$.

c.) $e^x = \frac{1}{3}$, so $x = \ln\left(\frac{1}{3}\right) = -\ln(3)$.

d.) $f(x) = \ln(1-x)$. Need $1-x > 0$, i.e. $x < 1$ D: $(-\infty, 1)$

e.) $g(s) = \frac{1}{\sqrt{s^2-1}}$ Need $s^2 - 1 > 0$, i.e. $s^2 > 1$ D: $(-\infty,-1) \cup (1,\infty)$

2. a.) $y = \sin(2x) + \frac{1}{2}$, $0 \le x \le \pi$

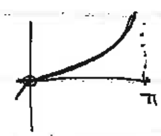


$y = 0$ when $\sin(2x) = -\frac{1}{2}$,

i.e. $2x = \frac{5\pi}{4}, \frac{7\pi}{4}$, so

$x = \frac{5\pi}{8}, \frac{7\pi}{8}$. No vertical asymptotes

b.) $y = \tan(x/2)$, $0 \le x \le \pi$

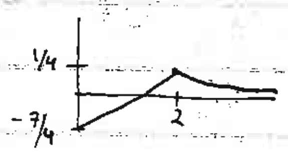


$\lim_{x\to\pi^-} \tan(x/2) = \lim_{y\to\pi/2^-} \tan(y)$

$= \lim_{y\to\pi/2^-} \sin(y)/\cos(y) \overset{1/+0}{=} +\infty$ $x = \pi$ vertical asymptote.

$y = 0$ when $\tan(x/2) = 0$, i.e. $\sin(x/2) = 0$. So $x/2 = 0$, i.e. $x = 0$.

3. a.)



b.) $\lim_{x\to2^+} h(x) = \lim_{x\to2^+} \frac{1}{x+2} = \lim_{x\to2} \frac{1}{x+2} = \frac{1}{4}$ and $h(2) = \frac{1}{4}$

$\lim_{x\to2^-} h(x) = \lim_{x\to2^-} \frac{4x-7}{4} = \lim_{x\to2} \frac{4x-7}{4} = \frac{1}{4}$

Yes, it's continuous on $(-\infty,\infty)$

c.) No. The secant lines from the left all have positive slope, =1, whereas from the right have negative slope. There's a corner at $x=2$.

4. a.) $\lim_{t\to 4} \frac{2t-8}{2-\sqrt{t}} = \lim_{t\to 4} \frac{2(t-4)}{2-\sqrt{t}} \cdot \frac{2+\sqrt{t}}{2+\sqrt{t}} = \lim_{t\to 4} \frac{2(t-4)(2+\sqrt{t})}{4-t} = \lim_{t\to 4} -2(2+\sqrt{t}) = -8$

b.) $\lim_{z\to\infty} \frac{2-\cos(2z)}{3z}$

$$1 \le 2 - \cos(2z) \le 3$$

$$\frac{1}{3z} \le \frac{2-\cos(2z)}{3z} \le \frac{3}{3z}$$

$$\lim_{z\to\infty} \downarrow 0 \quad = \quad \downarrow 0 \lim_{z\to\infty}$$

So $\lim_{z\to\infty} \frac{2-\cos(2z)}{3z} = 0$ by Squeeze Theorem

5. a.) $\lim_{x\to\infty} \frac{1+\sqrt{3x^6+1}}{x^3-1}$

$$\sqrt{x^6} = |x^3| = \begin{cases} x^3, & x \ge 0 \\ -x^3, & x < 0 \end{cases}$$

$= \lim_{x\to\infty} \frac{1+\sqrt{3x^6+1}}{x^3-1} \cdot \frac{1/\sqrt{x^6}}{1/x^3}$

$= \lim_{x\to\infty} \left(\frac{\frac{1}{\sqrt{x^6}} + \sqrt{3+\frac{1}{x^6}}}{1-1/x^3} \right) = \frac{0+\sqrt{3+0}}{1-0} = \sqrt{3}$

$\lim_{x\to-\infty} \frac{1+\sqrt{3x^6+1}}{x^3-1} = \lim_{x\to-\infty} \frac{1+\sqrt{3x^6+1}}{x^3-1} \cdot \frac{-1/\sqrt{x^6}}{1/x^3} = \lim_{x\to-\infty} \frac{-\frac{1}{\sqrt{x^6}} - \sqrt{3+\frac{1}{x^6}}}{1-1/x^3}$

$= \frac{-0-\sqrt{3+0}}{1-0} = -\sqrt{3}.$

Horizontal asymptotes: $y=\sqrt{3}$, $y=-\sqrt{3}$.

b.) $x^3-1=0$, $x^3=1$, i.e. $x=1$. $\lim_{x\to 1^-} \frac{1+\sqrt{3x^2+1}}{x^3-1} = \frac{3}{0^-} = -\infty$

Vertical asymptote: $x=1$. $\lim_{x\to 1^+} \frac{1+\sqrt{3x^2+1}}{x^3-1} = \frac{3}{0^+} = +\infty$

6. a.) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

b.) $f'(x) = \lim_{h\to 0} \frac{\frac{1}{x+8+h} - \frac{1}{x+8}}{h} = \lim_{h\to 0} \frac{\frac{x+8}{(x+8+h)(x+8)} - \frac{x+8+h}{(x+8)(x+8+h)}}{h}$

$= \lim_{h\to 0} \frac{\frac{-h}{(x+8)(x+8+h)}}{h} = \lim_{h\to 0} \frac{-1}{(x+8)(x+8+h)} = \frac{-1}{(x+8)^2}.$

7.) a.) $\frac{d}{dx}\left(f(2x)\right)\Big|_{x=1} = f'(2x)\cdot 2\Big|_{x=1} = 2\cdot f'(2) = 2\cdot 1 = 2.$

b.) $g'(x) = f'(x)\, e^{f(x)}$, so $g'(2) = f'(2)\, e^{f(2)} = 1\cdot e^{3} = e^{3}.$

8. a.) $\frac{d}{dx}\left[\frac{x\sin(x)}{x+1}\right] = \frac{(\sin(x) + x\cos(x))\cdot(x+1) - (x\sin(x))\cdot 1}{(x+1)^2}.$

b.) $\frac{d}{ds}\left[(s^{-2}+s^{2})^{30}\right] = 30(s^{-2}+s^{2})^{29}(-2s^{-3}+2s)$

c.) Taking derivatives, $2x - 3e^{-3y}y' = 4y' + 1$

Rearranging: $2x-1 = y'(4+3e^{-3y})$

So $y' = \frac{2x-1}{4+3e^{-3y}}$

At $(1,0)$: $y' = \frac{2\cdot 1-1}{4+3e^{-3\cdot 0}} = \frac{2-1}{4+3} = \frac{1}{7}.$

9. $g(1) = 3\tan(1-1) + 2\cos(1-1) = 3\tan(0) + 2\cos(0) = 3\cdot 0 + 2\cdot 1 = 2.$

$g'(x) = 3\sec^2(x-1)\cdot 1 - 2\sin(x-1)\cdot 1$

$= 3\sec^2(x-1) - 2\sin(x-1)$

$g'(1) = 3\sec^2(1-1) - 2\sin(1-1) = 3\cdot\sec^2(0) - 2\sin(0) = 3\cdot 1^2 - 2\cdot 0 = 3.$

$y - g(1) = g'(1)(x-1) \rightsquigarrow y - 2 = 3(x-1).$

10. a.) False - can't tell that from $f'(t)$.

b.) True - $f'(t)$ is negative from $t=3$ to $t=4$, so water level dropping.

c.) True - the slope of the tangent line at $t=2$ is $f'(2)=0$.

d.) False - $\frac{f(3.1)-f(3)}{.1} \approx f'(3)$, which is negative.

7.) a.) $\frac{d}{dx}\left(f(2x)\right)\Big|_{x=1} = f'(2x)\cdot 2\Big|_{x=1} = 2\cdot f'(2) = 2\cdot 1 = 2.$

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$g'(x) = 3\sec^2(x-1)\cdot 1 - 2\sin(x-1)\cdot 1$

$= 3\sec^2(x-1) - 2\sin(x-1)$

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