

WEIGHTED VOTING HANDOUT SOLUTIONS

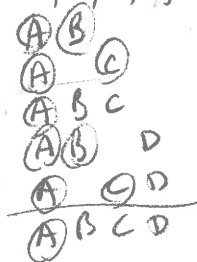
1 a) [15: 16, 8, 4, 1]

since $16 >$ quota, A is a dictator

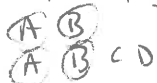
b) [18: 16, 8, 4, 1]

A has veto, D is a dummy, as shown in coalitions at the right \rightarrow

[18: 16, 8, 4, 1]



c) [24: 16, 8, 4, 1]



A and B both have vetos
C and D both are dummies

2) [10: 6, 5, 4, 2]

a) AC has weight $6+4 = 10$

b) (A, B)

(A, C)

(A, B, C)

(A, B, D)

(A, C, D)

(B, C, D)

A B C D

c) (A, B, C) only A is critical

d) critical players circled in part b.

$\beta_A = \frac{5}{12} \quad \beta_B = \beta_C = \frac{3}{12} \quad \beta_D = \frac{1}{12}$

3) [10: 5, 4, 3, 2, 1]



(A, B, C, D)

(A, B, C, E)

(A, B, D, E)

(A, C, D, E)

(B, C, D, E)

A B C D E

[11: 5, 4, 3, 2, 1]



(A, B, C, D)

(A, B, C, E)

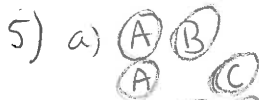
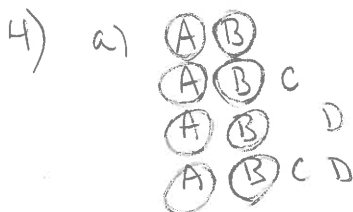
(A, B, D, E)

(A, C, D, E)

(A, B, C, D, E)

$\beta_A = \frac{8}{24} \quad \beta_B = \frac{6}{24} \quad \beta_C = \beta_D = \frac{4}{24} \quad \beta_E = \frac{2}{24}$

$\beta_A = \frac{7}{19} \quad \beta_B = \frac{5}{19} \quad \beta_C = \beta_D = \frac{3}{19} \quad \beta_E = \frac{1}{19}$



A B C

(A, B, D)

(A, C, D)

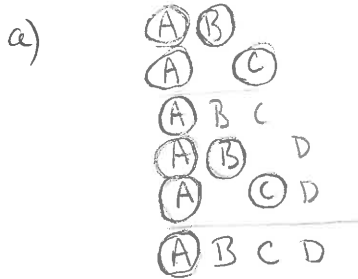
(B, C, D)

A B C D

$\beta_A = \beta_B = \beta_C = \frac{4}{12} = \frac{1}{3}$
 $\beta_D = 0$

$\beta_A = \beta_B = \frac{4}{8} = \frac{1}{2}$
 $\beta_C = \beta_D = 0$

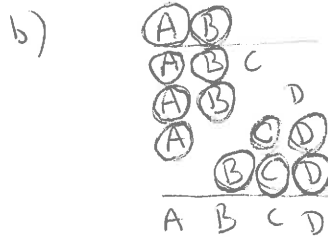
6.) [8: 6, 4, 2, 1]



$$\beta_A = \frac{6}{10} \quad \beta_B = \beta_C = \frac{2}{10} \quad \beta_D = 0$$

D buys a vote from A:

[8: 5, 4, 2, 2]

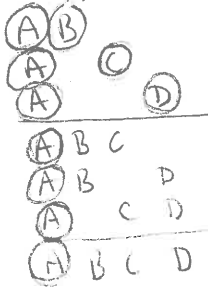


$$\beta_A = \frac{4}{12} = \beta_B \quad \beta_C = \beta_D = \frac{2}{12}$$

notice power for A ↓, B ↑, C ↓, D ↑

c) D buys a vote from B

[8: 6, 3, 2, 2]



$$\beta_A = \frac{7}{10} \quad \beta_B = \beta_C = \beta_D = \frac{1}{10}$$

notice power for A ↑, B ↓, C ↓, D ↑

d) D buys a vote from C:

[8: 6, 4, 1, 2]

this is the same as original with the last two players swapping roles

$$\beta_A = \frac{6}{10} \quad \beta_B = \frac{2}{10} \quad \beta_C = 0, \quad \beta_D = \frac{2}{10}$$

So power for A, B same as original for C ↓, D ↑

e) C loses power in all three scenarios!

D gets the most power by buying from C, though since C would lose all power, C might be extremely reluctant to sell.

D gets second most power by buying from A, but since by selling A loses not only some power but also a neto, A might also be reluctant to sell.

D gets a little power by buying from B, and B loses a little.

7.) a) [27: 10, 8, 6, 4, 2]

A B C D
A B C D E

$$\beta_A = \beta_B = \beta_C = \beta_D = \frac{1}{4} \quad \beta_E = 0$$

c) E buys a vote from B

[27: 10, 7, 6, 4, 3]

A B C D
A B C D E

no change from original

e) E buys a vote from D

[27: 10, 8, 6, 3, 3]

A B C D
A B C E
A B C D E

$$\beta_A = \beta_B = \beta_C = \frac{3}{11}$$

$$\beta_D = \beta_E = \frac{1}{11}$$

b) E buys a vote from A.

[27: 9, 8, 6, 4, 3]

A B C D
A B C D E

no change from original

d) E buys a vote from C

[27: 10, 8, 5, 4, 3]

A B C D
A B C D E

no change from original

f) Even though D charges more for a vote,
E should buy from D.

$$\text{D loses } \frac{3}{12} - \frac{1}{11} \approx .16$$

$$\text{E gains } \frac{1}{11} \approx .09$$

but notice A, B, and C each gain power, too...

$$\frac{3}{11} - \frac{3}{12} \approx .023$$

So, perhaps E could get A, B and C to help finance E's purchase of a vote from D.

- 8.) [22: 10, 10, 10, 10, 1] a) Player E is a dummy
 b) $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 1/4$ $\beta_5 = 0$

- 9.) [34: 10, 10, 10, 10, w]
 player E is a dummy when $w = \boxed{0, 1, 2, 3}$

- 10.) [q: 10, 10, 10, 10, 1]
 player E is not a dummy when $q = \boxed{21, 31, 41}$

11.) faculty: A, B students: C, D, E

a) there are $2^5 - 1 = 32 - 1 = 31$ coalitions possible

b) coalitions of size 3: ABC ACD BCD
 ABD ACE BCE
 ABE ADE BDE

all players critical in all coalitions

c) coalitions of size 4: ABCD
 ABC E
 AB DE
 (A) CDE
 (B) CDE

d) grand: ABCDE no one critical

e) $\beta_A = \beta_B = 7/29$ $\beta_C = \beta_D = \beta_E = 5/29$

- 12.) [q: 8, 4, 2] a) total votes = 14 so $14/2 < q$, so $7 < q \rightarrow \boxed{q = 8}$

b) largest q is $\boxed{14}$

c) A, B, and C have veto if $q \geq 8+4+1$, so $q = \boxed{13 \text{ or } 14}$

d) B has veto and not C if $q = \boxed{11 \text{ or } 12}$

e) C is the only dummy if $q = \boxed{11 \text{ or } 12}$

13) 1a) [1: 1, 0, 0, 0] 1c) [2: 1, 1, 0, 0] 2) [5: 3, 2, 2, 1]

3a) [8: 4, 3, 2, 2, 1] 3b) [9: 4, 3, 2, 2, 1] 4) [2: 1, 1, 0, 0]

5) [2: 1, 1, 1, 0] 6a) [3: 2, 1, 1, 0] 6b) [4: 2, 2, 1, 1] 6c) [4: 3, 1, 1, 1]

7a) [4: 1, 1, 1, 1, 0] 7e) [7: 2, 2, 2, 1, 1] 8) [3: 1, 1, 1, 1, 0] 11) [7: 3, 3, 2, 2, 2]

notice [3: 2, 1, 1] is not equivalent, since BCD should not win.

[5: 2, 2, 1, 1, 1] is not equivalent, since ACD should win.
 [7: 3, 3, 1, 1, 1] is not equivalent, since ACD should win.

notice also