

Weighted Voting Handout

- 1) For each of the following systems, decide which players (if any) are dictators, dummies, or have veto power.
 - a) $[15 : 16, 8, 4, 1]$
 - b) $[18 : 16, 8, 4, 1]$
 - c) $[24 : 16, 8, 4, 1]$
- 2) Consider the weighted voting system $[10 : 6, 5, 4, 2]$.
 - a) What is the weight of coalition AC?
 - b) Find all winning coalitions.
 - c) Which players are critical in the coalition ABC?
 - d) Find the Banzhaf power distribution.
- 3)
 - a) Find the Banzhof power distribution for $[10 : 5, 4, 3, 2, 1]$.
 - b) Use your work from part a) to help you find the Banzhaf power distribution for $[11 : 5, 4, 3, 2, 1]$... notice only q has changed.
- 4) In a weighted voting system with four players, the winning coalitions are AB, ABC, ABD, ABCD.
 - a) Find the critical players in each coalition.
 - b) Find the Banzhaf power distribution.
- 5) A law firm is run by four partners, each having one vote. Decisions are made by majority rule, but in the case of a 2-2 tie, the coalition with D (the junior partner) loses.
 - a) Find the critical players in each coalition.
 - b) Find the Banzhaf power distribution.
- 6) You are partner D in the weighted voting system $[8 : 6, 4, 2, 1]$. You offer to buy a vote, and the other three partners each offer one for the same price.
 - a) Find the Banzhaf power distribution for the original system.
 - b) Find the Banzhaf power distribution if you buy a vote from A.
 - c) Find the Banzhaf power distribution if you buy a vote from B.
 - d) Find the Banzhaf power distribution if you buy a vote from C.
 - e) Who should you buy a vote from?

- 7) You are partner E in the weighted voting system $[27 : 10, 8, 6, 4, 2]$. You offer to buy a vote, and A, B, and C each offer you one for \$1000, but D wants \$5000 for one.
- Find the Banzhaf power distribution for the original system.
 - Find the Banzhaf power distribution if you buy a vote from A.
 - Find the Banzhaf power distribution if you buy a vote from B.
 - Find the Banzhaf power distribution if you buy a vote from C.
 - Find the Banzhaf power distribution if you buy a vote from D.
 - Who should you buy a vote from?
- 8) Consider the weighted voting system $[22 : 10, 10, 10, 10, 1]$.
- What players are dummies?
 - Without listing any coalitions, find the Banzhaf power distribution.
- 9) Consider the weighted voting system $[34 : 10, 10, 10, 10, w]$. Find all integer values for w that will make player E a dummy.
- 10) Consider the weighted voting system $[q : 10, 10, 10, 10, 1]$. Find all integer values for q that will make player E not a dummy.
- 11) The judicial board of a university is composed of five members, two of which must be faculty (call them A and B), and three of which must be students (call them C, D, and E). To pass a motion requires at least three votes, and at least one of these votes must be from a faculty member.
- How many total coalitions are there in this system?
 - List all the winning coalitions of size 3, and circle the critical players.
 - List all the winning coalitions of size 4, and circle the critical players.
 - List all the winning coalitions of size 5, and circle the critical players.
 - Find the Banzhaf power distribution.
- 12) Consider the weighted voting system $[q : 8, 4, 2]$.
- What's the smallest value possible for q ?
 - What's the largest value possible for q ?
 - What values of q make all three players have veto power?
 - What values of q make B have veto power, but not C?
 - What values of q make C the only dummy?
- 13) Find the minimal systems for the weighted voting systems in problems 1a, 1c, 2, 3a, 3b, 4, 5, 6a, 6b, 6c, 7a, 7e, 8, 11.