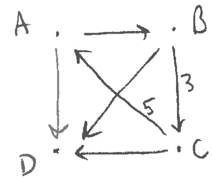
Chapter 5 homework solutions (day 2)

1)

1	2	1	1	2	2
D	D	D	A	B	C
A	B	C	B	C	A
B	C	A	C	A	B
C	A	B	D	D	D

A : B 5:4
A : C 2:7
A : D 5:4
B : C 6:3
B : D 5:4
C : D 5:4



S = {A, B, C}

a) a posteriori Smith fair plurality: A = 1 B = 2 C = 2 D = 4 $\boxed{W = \{B, C\}}$

b) a priori Smith fair plurality: drop D

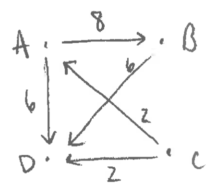
1	2	1	1	2	2
A	B	C	A	B	C

A = 2 B = 4 C = 3 $\boxed{W = \{B\}}$

2)

2	2	1	3	2
A	A	B	C	D
B	B	C	A	C
D	C	A	B	A
C	D	D	D	B

A : B 9:1
A : C 4:6
A : D 8:2
B : C 5:5
B : D 8:2
C : D 6:4



S = {A, B, C}

a) Elimination : elim B,

2	2	1	3	2
A	A	C	C	D

elim D, A : C W = {C}

but if we drop D,

2	2	1	3	2
A	A	B	C	C

elim B, A : C W' = {C} $\boxed{\text{D is not a spoiler}}$

b) Borda:

A = 0 + 6 + 9 + 16 = 31
B = 2 + 6 + 12 + 4 = 24
C = 2 + 4 + 9 + 12 = 27
D = 6 + 4 + 0 + 8 = 18
100 = 10·10 ✓

W = {A}.

but if we drop D,

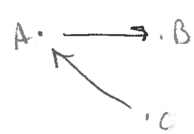
2	2	1	3	2
A	A	B	C	C
B	B	C	A	A
C	C	A	B	B

A = 1 + 10 + 12 = 23
B = 5 + 8 + 3 = 16
C = 4 + 2 + 15 = 21
60 = 6·10 ✓

W' = {A} $\boxed{\text{D is not a spoiler}}$

c) Pairwise: A = 2, B = $1\frac{1}{2}$, C = $2\frac{1}{2}$, D = 0 total = 6 ✓
W = {C}

but if we drop D,



A = 1
B = $\frac{1}{2}$
C = $1\frac{1}{2}$

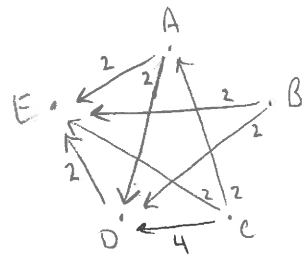
W' = {C} $\boxed{\text{D is not a spoiler}}$

Chapter 5 homework solutions (day 2) continued

6)

1	1	1	1
A	E	C	C
B	B	B	A
C	C	D	B
D	A	A	D
E	D	E	E

A : B 2 : 2
A : C 1 : 3
A : D 3 : 1
A : E 3 : 1
B : C 2 : 2
B : D 3 : 1
B : E 3 : 1
C : D 4 : 0
C : E 3 : 1
D : E 3 : 1



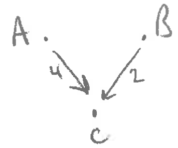
Sequential winner: C

A B C D E
↑ · ↑ · ·

$S = D_C = \{C, B, A\}$

7)

2	1	1
B	A	A
A	B	C
C	C	B



a) sequential winner A B C $S = D_A = \boxed{\{A, B\}}$
↑ · ·

b) a posteriori Smith fair Borda

$A = 0+4+6 = 10$
$B = 1+2+6 = 9$
$C = 3+2+0 = 5$
$24 = 4\cdot 6$ ✓

$\boxed{W = \{A\}}$

c) a priori Smith fair Borda. drop C,

A : B 2 : 2 $\boxed{W = \{A, B\}}$

8) Prove Pairwise is a priori Smith fair.
We need to show the presence or absence of NSC's doesn't affect W.
If a NSC were dropped from the p.s, what would happen? Since

we know every SC would lose a pairwise point. Since $W \subset S$, and Since this happens to every SC, W will not change.

9) Prove Sequential is a priori Smith fair.
We need to show the presence or absence of NSC's doesn't affect W.
Once the candidates are in a fixed order, we know the 1st SC will beat all the NSC's who came before.

NSC NSC NSC SC __ __ __
↑

If any of these NSC's are dropped,
it doesn't matter, the 1st SC will still beat previous candidates.
Once a SC is the current contender, no NSC can beat them,

So __ __ SC NSC NSC NSC
↑ · · ·

So if any of these NSC's are dropped,
it won't matter.

Once a SC is the current contender, only another SC can beat them.