

Chapter 3 homework solutions

1) $\frac{2 \ 4 \ 3}{C \ B \ A}$ A:B 5:4 A $\xrightarrow{1}$ B
 A C B A:C 3:6 ↙ 3 ↘ 5
 B A C B:C 7:2 C

a) Plurality $w = \{B\}$.
 drop C from preference schedule
 $\frac{5 \ 4}{A \ B}$ $w' = \{A\}$.
 B A

b) runoff A=B $w = \{A\}$
 drop B from preference schedule
 $\frac{6 \ 3}{C \ A}$ runoff A+C $w' = \{C\}$
 A C

c) eliminate C, A:B $w = \{A\}$.
 drop B from preference schedule,
 $\frac{6 \ 3}{C \ A}$ $w' = \{C\}$
 A C

d) Coombs: eliminate A, $\frac{2 \ 7}{C \ B}$ $w = \{B\}$. But drop C from the
 B C
 preference schedule, so it becomes $\frac{5 \ 4}{A \ B}$ $w' = \{A\}$.
 B A

e) Borda: A = 4 + 4 + 9 = 17
 B = 2 + 6 + 12 = 20
 C = 3 + 8 + 6 = 17
 54 = 6.9 ✓ $w = \{B\}$.
 but drop C from preference schedule,
 so $\frac{5 \ 4}{A \ B}$ $w' = \{A\}$.
 B A

f) Pairwise comparison $A=B=C=1$ $w = \{A, B, C\}$.
 but drop any candidate to find a violation.
 for example, drop A, $\frac{2 \ 7}{C \ B}$ and $w' = \{B\}$
 B C

g) sequential winner A B C $w = \{C\}$
 ↑ ↑
 But drop A from preference schedule B C $w' = \{B\}$.
 ↑

- 2) See work in number 1 above.
- a) C is a losing spoiler in plurality.
 - b) B is a losing spoiler in runoff.
 - c) B " " " " elimination
 - d) C " " " " Coombs
 - e) C " " " " Borda
 - f) A " " " " sequential comparison.

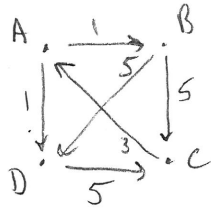
Notice in number 1 above f) A is a winning spoiler. (B and C are also winning spoilers, which you can show).

Chapter 3 homework solutions continued

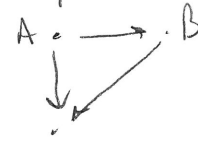
3) There are many examples. I'll make one by adding candidate D to the election in number 1 above.

2	4	3
C	B	A
A	D	B
D	C	D
B	A	C

A:B 5:4
 A:C 3:6
 A:D 5:4
 B:C 7:2
 B:D 7:2
 C:D 2:7



Pairwise $A=B=2$ $C=D=1$ so
 but drop C $w = \{A, B\}$



$A=2$ $B=1$
 $w' = \{A\}$

so C is a losing spoiler
 (it's also possible to show D is a losing spoiler)

4)

4	3	4	2
B	A	C	C
A	B	A	B
C	C	B	A

A:B 7:6
 A:C 7:6
 B:C 7:6

a) A beats B and C, so A is Condorcet

b) Plurality $w = \{C\}$.

drop A,

7	6
B	C
C	B

$w' = \{B\}$.

so A is a losing spoiler.