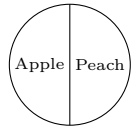


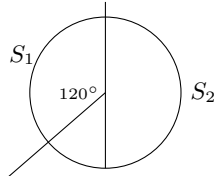
# Fair Division Handout

1. Two people are dividing a pie that is half apple and half peach using "I cut, you choose".

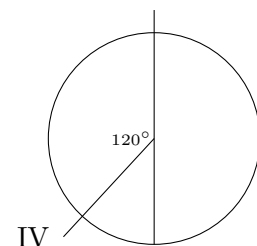
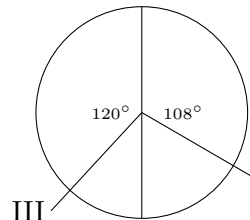
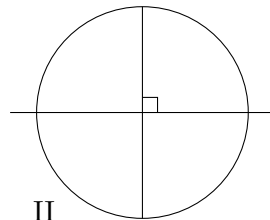
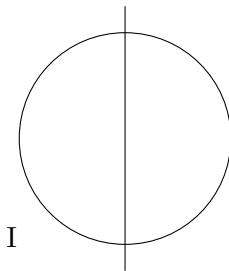
J likes apple three times as much as peach, and K likes apple twice as much as peach.



Consider the following cut:



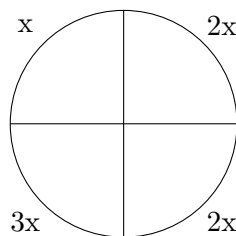
- (a) If J was the divider, would J make that cut?
- (b) What fraction of the pie is each piece worth to K?
- (c) What fair division results?
- (d) Is the above division equitable?
- (e) Is the above division envy-free?



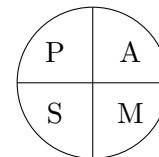
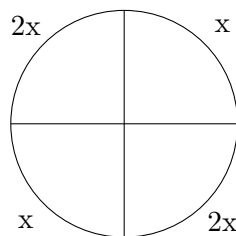
- (f) If J was the divider, list all of the above cuts that J could possibly make.
- (g) If K was the divider, list all of the above cuts that J could possibly make.

2. Two people are using "I cut, you choose" to divide a pizza that has pepperoni, anchovies, mushroom and sausage.

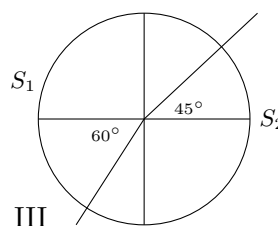
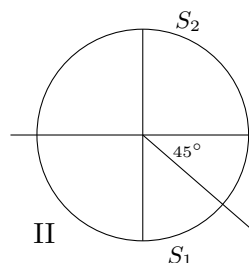
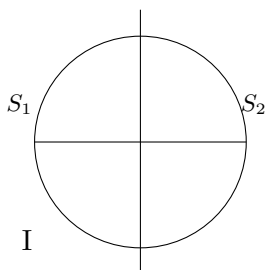
D's preferences



C's preferences

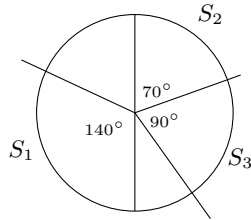


(a) Show that these are three different cuts that D could make as the divider:



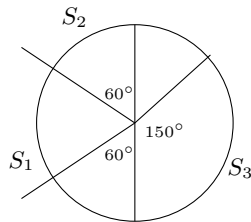
(b) Now for each different cut, find the value of the two pieces to C, and find the division that results.

3. Three people (B, C and D) are using the Steinhaus method to divide a pie that is half apple (left) and half peach (right). B likes both equally, C hates apple and loves peach, and D likes peach twice as much as apple.



If D divides the pie into three pieces as shown above, make a chart showing the fraction of the pie that each piece is worth to each player, find the bid lists, and then find a fair division that results.

4. Three people (B, C and D) are using the Steinhaus method to divide a pie that is half apple (left) and half peach (right). B hates apple and loves peach. C likes peach twice as much as apple, and D loves apple and hates peach.



If D divides the pie into three pieces as shown above, make a chart showing the fraction of the pie that each piece is worth to each player, find the bid lists, and then find a fair division that results.

5. Five players (A, B, C, D and E) are using the Steinhaus method to divide a lot of land. The lot is divided into five pieces by D, and the table below summarizes the values of each piece to each player:

	$S_1$	$S_2$	$S_3$	$S_4$	$S_5$
$A$	$\frac{9}{50}$	$\frac{15}{50}$	$\frac{8}{50}$	$\frac{10}{50}$	$\frac{8}{50}$
$B$	$\frac{9}{50}$	$\frac{9}{50}$	$\frac{14}{50}$	$\frac{9}{50}$	$\frac{9}{50}$
$C$	$\frac{2}{50}$	$\frac{14}{50}$	$\frac{10}{50}$	$\frac{12}{50}$	$\frac{12}{50}$
$D$	$\frac{10}{50}$	$\frac{10}{50}$	$\frac{10}{50}$	$\frac{10}{50}$	$\frac{10}{50}$
$E$	$\frac{9}{50}$	$\frac{15}{50}$	$\frac{8}{50}$	$\frac{9}{50}$	$\frac{9}{50}$

- (a) List the pieces each player would bid on if they bid honestly, and then describe a fair division.
- (b) If player C dishonestly bids on  $S_2$ , find a division that could result which is fair to A, B and D, but not fair to C (be sure to show why it can be unfair to C, yet must be fair to the others).

6. Three people are using Selfridge-Conway to divide up a berry tart that has the following configuration:

L	M	M
H	H	M
H	H	M
H	H	M

Suppose A likes all three types of tart equally, B likes H and M equally, but likes L four times as much as those, and C likes L and H equally, but likes M twice as much. Also, suppose A divides the tart into three pieces by the columns, so we can call the pieces left, middle, and right.

- What piece would B choose to trim?
  - Now, assume B trims off the bottom of that piece, so that they are now the trimmings. Complete the Selfridge/Conway division, and describe what each person ends up getting.
  - Find how much each person thinks each other person got, and verify this is an envy-free division.
7. Three people are using Selfridge-Conway to divide up a berry tart that has the following configuration:

M	M	L
M	M	L
M	M	L
M	M	H
H	M	H

Suppose A likes all three types of tart equally, B likes L and M equally, but likes H four times as much as those, and C likes H, but likes L three times as much, and C hates M. Also, suppose A divides the tart into three pieces by the columns, so we can call the pieces left, middle, and right.

- What piece would B choose to trim?
  - Now, assume B trims off the top of that piece, so that they are now the trimmings. Complete the Selfridge/Conway division, and describe what each person ends up getting.
  - Find how much each person thinks each other person got, and verify this is an envy-free division.
8. Say A and B jointly own a painting, and one of them will keep it. A thinks it's worth \$1200, and B thinks it's worth \$800.
- Describe all possible fair and envy-free arrangements.
  - Find an equitable compensation arrangement.
  - Use the Adjusted Winner method to find an equitable division if each puts \$600 in a joint account.
  - Repeat part c, but have each player put \$800 in the joint account.
  - Repeat part c, but have each player put \$2400 in the joint account.
  - Compare the percent each player gets in the answers to parts b, c, d and e.