## Compensation Applet Worksheet

Click on the applet on the course website (it is below the *Resources* section). The first time you open the applet, you may be prompted to download the CDF player from Wolfram. Once the applet is open, you should click on the arrows to the left of *Point location* and *Values of the object*, to expand those menus. Then, under "Values", for each player, expand the view of their bid by clicking on the plus at the end of the slidebar. There are two more menus for *Point Increments* and *Plot Options*, and you may expand these if you'd like (but I don't think you'll need them for this worksheet).

Note that for all work in the applet, A gets the object, and B and C each get cash payouts. B's payout is represented on the horizontal axis, and C's payout is represented on the vertical axis. Some of the questions here can just be answered by looking at the applet, but some will require you to do some computation. Use the menus to the left to set A's bid to 60 , B's bid to 45, and C's bid to 30. The fairness triangle should appear. It divides all possible compensation arrangements into 7 areas, as shown here:
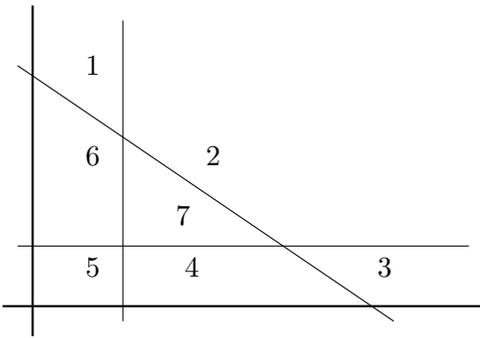
1. Click and drag the dynamic point (which looks a little like: ⊕) to a spot in any of the 7 regions, and fill in the chart below. The coordinates of the dynamic point appear under *Point location*.

Area	X_B	X_C	compute X_A	fair to whom?	specify all instances of envy in this arrangement

2. Now move the dynamic point to a spot in another region, and fill in the chart below:

Area	X_B	X_C	compute X_A	fair to whom?	specify all instances of envy in this arrangement

3. Move the dynamic point to yet another spot in another region, and fill in the chart below:

Area	X_B	X_C	compute X_A	fair to whom?	specify all instances of envy in this arrangement

4. We know envy-free implies fair, for any N. This is logically equivalent to unfair implies envy. How does that make sense with the fairness triangle? List which regions should have only arrangements in which there is envy.

Does this agree with your work in the tables above?

5. Now, there are 4 regions you have not explored. In the chart below, for each of those regions, you should predict who will get fair shares, and decide if envy-free arrangements are possible in that region.

Area	fair to whom?	is envy-free possible in this region?

6. Now set $b = c = 30$, and move the dynamic point to (30,30). Using the slide bar, increase a until you can see that the arrangement with $X_B = X_C = 30$ is on the boundary of the fairness triangle. What's the smallest value of a that makes this possible? Is this an envy-free arrangement?

7. Now click on the box next to *Equal compensation amounts*. A green line should appear. How does this line relate to the envy-free arrangements that are possible?

Do you think this is an equitable arrangement?

8. The applet tells you what q is for an equitable arrangement. Verify q is correct, and compute the three payouts.

9. Now click on the box next to *Equitable arrangement*. A black dot should appear. Does this agree with the equitable arrangement you found?

The remaining questions are optional, and are intended to help you explore how the bids, the payouts, and the arrangements are inter-related. If you have time to do them, or if you think of other ways to use the applet, I'd be interested in hearing your feedback.

10. Set a = 60, b = 45, and c = 30. Activate the Equitable arrangement by clicking the box next to it. Do the same for the Equal compensation arrangements.

    Slide b to the left until the equitable arrangement is on the green line, $X_B = X_C$. What must be true for B's and C's bids for this to happen?

    Slide b back to 45. Then try sliding a to the far right and far left. Geometrically, what is happening to the fairness triangle and the equitable arrangement? Which equitable arrangements are envy-free? What's the smallest value of a that makes the equitable arrangement envy-free? When the value of a is smaller than that, who has envy?

11. Next set $a = 60, b = c = 30$, and make sure the equitable and equal compensation boxes are checked. Slide A's bid around... what's the smallest a can be to make a fair arrangement?

    Compare the q value for this min value of A's bid, to the q value when A's bid is 90.

12. Set a = 30 and c=30. Slide b around, and notice what happens to the fairness triangle. What values of b make a fair arrangement possible or impossible? Why is this happening? How does this relate to m, the average bid?

13. Set $a = 60, b = 40, c = 30$. Move the dynamic point to (20,20). Activate the Equal Compensation Amounts line. The point (20, 20) is the intersection of two lines on the graph. Which two lines? Circle two:

$$X_A = \frac{a}{3} \qquad\qquad X_B = \frac{b}{3} \qquad\qquad X_C = \frac{c}{3} \qquad\qquad X_B = X_C$$

Slide b to the right. Which values of b make the compensation arrangement $X_B = X_C = 20$ fair?

There is a value of b which makes only one envy-free arrangement possible. What is that value?

For what range of values of b is an envy-free arrangement possible? Specify the lowest value of b and the highest value of b.