Prove that the Steinhaus method is always fair for N = 3

Proof 3:

Easy case: A, B, C all make a bid list. N = 3. Each player is then given a piece from their bid list (which is composed of preferences that would give a fair share).

Hard case: B and C both only want S_1. We give A S_2 or S_3, say S_3, and have A and B do I cut you choose on the recombined S_1+ S_2. A and B both value $S_1 > 2/3$, because ~~S_2 and~~ S_3 would not be fair to them. When they perform I cut you choose, each is guaranteed at least 1/2 of 2/3 which is 1/3, a fair division.

should say B and C
both value $(S_1+S_2) > 2/3$
since S_3 worth $< 1/3$, (S_1+S_2) worth $> 1/3$

Proof 5:

Easy case: Everyone gets a piece on their bid list.

Hard case: Let's say B and C both want S_1 and only S_1. Let's say D wants S_2 (or S_3). D gets S_2 - fair share. B and C believe $S_1 > 1/3$ $S_2 < 1/3$ $S_3 < 1/3$. Recombine S_1 and S_3 (or S_2) and divide - "I cut, you choose". Both B and C will get half of piece that they believe is > 1/3 - so it is fair to B and C too.

D lists all 3 on bidlist, so "wants" them all.
We can choose to give D S_2.

Proof 7:

Easy case: Each player get piece of their bid list, so each gets at least 1/3. Fair.

Hard case: - D divides 3 equal pieces, guaranteed to get at least 1/3.

- B and D both only value S_1
- This means they see $S_1 > 2/3$, while S_2 & S_3 both <1/3.
- Each guaranteed 1/2 of 2/3, which is 1/3.
- Therefore, fair.

D gets exactly a fair share
Should say B and C
They only value $S_1 > 1/3$, not 2/3
Why? need to give a piece to D, describe recombined piece, and its worth, and then mention "I cut you choose", which is what guarantees 1/2 to each.