

Prove that the Steinhaus method is always fair for $N = 3$

Find the errors in these three proofs from former students:

Proof 3:

Easy case: A, B, C all make a bid list. $N = 3$. Each player is then given a piece from their bid list (which is composed of preferences that would give a fair share).

Hard case: B and C both only want S_1 . We give A S_2 or S_3 , say S_3 , and have A and B do I cut you choose on the recombined $S_1 + S_2$. A and B both value $S_1 > 2/3$, because S_2 and S_3 would not be fair to them. When they perform I cut you choose, each is guaranteed at least $1/2$ of $2/3$ which is $1/3$, a fair division.

Proof 5:

Easy case: Everyone gets a piece on their bid list.

Hard case: Let's say B and C both want S_1 and only S_1 . Let's say D wants S_2 (or S_3). D gets S_2 - fair share. B and C believe $S_1 > 1/3$ $S_2 < 1/3$ $S_3 < 1/3$. Recombine S_1 and S_3 (or S_2) and divide - "I cut, you choose". Both B and C will get half of piece that they believe is $> 1/3$ - so it is fair to B and C too.

Proof 7:

Easy case: Each player get piece of their bid list, so each gets at least $1/3$. Fair.

Hard case: - D divides 3 equal pieces, guaranteed to get at least $1/3$.

- B and D both only value S_1
- This means they see $S_1 > 2/3$, while S_2 & S_3 both $< 1/3$.
- Each guaranteed $1/2$ of $2/3$, which is $1/3$.
- Therefore, fair.