

Proofs

Here are four ‘proofs’ of the statement “If a division is envy free, then it is fair.” Can you find what is wrong with each of them?

1. Since the division is envy free, A in particular has no envy. Since A has no envy, A is satisfied. Similarly, B has no envy, B is satisfied. This is true for all the participants, so it is fair to everyone.
2. Since the division is envy free, A does not want anyone else’s slice. In other words, A is satisfied. This must be fair to A . B also thinks B got a largest slice, B is satisfied. This is true for all the participants, so it is fair to everyone.
3. Proof by contrapositive. The contrapositive of the statement is “If a division is unfair, then there is envy.” Since the division is unfair, there is envy. But if every candidate gets less than $\frac{1}{N}$, there is no envy. So there must be envy.
4. Proof by contrapositive. Since the division is unfair, it is unfair to A . And A is not satisfied. Therefore A envies whoever did.