

MATH 19-01: PRACTICE PROBLEMS FOR FINAL EXAM

TUFTS UNIVERSITY DEPARTMENT OF MATHEMATICS
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VOTING SYSTEMS

- (1) Refer to review materials for exams 1 and 2 on the course website. Use the Voting Handout for extra practice. (We covered everything on the handout except “polarizing candidates.”)

- (2) Consider the following preference schedule:
- | | | | |
|------------|------------|------------|------------|
| $\times 4$ | $\times 3$ | $\times 4$ | $\times 2$ |
| B | A | C | C |
| A | B | A | B |
| C | C | B | A |

Verify that A is a Condorcet candidate and find a voting method for which A is a losing spoiler.

- (3) On a pairwise comparison graph with 55 voters, explain why all of the margins of victory must be odd.

APPORTIONMENT

- (4) For apportionment, M denotes population and m denotes number of seats. The i th state or district has a quota $Q_i = \frac{M_i}{M} \cdot m$. Explain the terms in that formula and what the quota means.

GEOMETRY OF GERRYMANDERING

- (5) Recall that the compactness score of a shape is $C(S) = \frac{400\pi A}{P^2}$. Showing all work, verify that for a regular hexagon H , the score is $C(H) = \frac{50\pi\sqrt{3}}{3}$.
- (6) The first Gingles factor requires that a minority population be “sufficiently large and geographically compact to constitute a majority in a single-member district.” Ohio’s population was 12.04% black on the 2010 census. How many congressional seats must it get in order that the black population can possibly pass this Gingles test? (For instance, if Ohio only gets one seat, then 12.04% is not a majority, so it would fail this Gingles factor.)

PROBABILITY AND ELECTIONS

- (7) Consider a pairwise comparison graph with random arrows (no ties). Find the probability that there is a Condorcet candidate if the number of candidates is 3, 4, 5 and n .
- (8) Suppose that we are going to hold a sequential election with 6 candidates, M, A, T, H, S, C .
- How many sequences are possible?
 - If all sequences are equally likely, what is the probability that the sequence ends up in alphabetical order?
 - What is the probability that M, A are the first two candidates in the sequence?
- (9) In a particular election, 3% of ballots are spoiled. Let’s suppose that election workers assign spoiled ballots randomly to D or R. Let’s also suppose that out of the clear ballots, election workers have a 98% chance of tallying the ballot correctly and the remaining 2% of the time they assign it randomly to D or R. Finally, let’s assume that the voter intent was for 43% to vote D and 57% to vote R. Make a decision tree showing all of the possible outcomes, and explain how to calculate the predicted vote totals for D and R in this model.

Answers will be provided on request in person at the review session on Thursday.