MATH 19-01: HW 9

We've defined the Isoperimetric Index (I.I.) of a shape to be the ratio P^2/A, where P is the perimeter and A is the area of the shape.

Now for any shape S, let's define its *compactness score* $C(S) = \dfrac{400\pi A}{P^2}$.

(1) (a) Verify that the I.I. of a circle of radius 10 is the same as for a circle of radius 3. Going further, verify that the I.I. of a circle of radius r does not depend on r.

(b) Verify that the I.I. of a square with a side of length s does not depend on s.

(c) Suppose that a rectangle has length ℓ and width w. Show that the I.I. only depends on the ratio ℓ/w.

(2) First show that the $C(S)$ formula is equivalent to $\dfrac{100 \cdot II(\text{circle})}{II(S)}$.
Now, using the isoperimetric theorem, prove that $0 \leq C(S) \leq 100$ for all shapes S.

(3) Find the compactness scores of the shapes from the last part (circle of radius r, square of side s, and rectangle of sides ℓ and w).

(4) Let H be a regular hexagon, and let H' be a hexagon with vertices $(1,0),(1,1),(0,1),(-1,0),(-1,-1),(0,-1)$. Let O a regular octagon and O' an octagon with vertices $(2,1),(1,2),(-1,2),(-2,1),(-2,-1),(-1,-2),(1,-2),(2,-1)$. Sketch these shapes, f ind their compactness scores, and make a conjecture about which polygons are the most "compact."

(5) (a) The original gerrymander! Right here in Massachusetts. This is a famous political cartoon from 1812 objecting to the shape of the South Essex district in the MA legislature, designed to favor Governor Gerry's favored candidates. Estimate its compactness score and explain how you do so.

(b) Same for this Texas congressional district.

TEXAS CONGRESSIONAL DISTRICT 25