

MATH 19-01: HW 8

(1) (a) Suppose there are $m = 11$ seats on a governing council, and the districts in the town (districts B , P , and R) have populations $M_B = 54$, $M_P = 243$, and $M_R = 703$. Apportion the seats by Hamilton's method. (There's no constitutional requirement that everybody gets a seat on the council.)

(b) In the following election cycle, the populations have grown a bit to $M'_B = 56$, $M'_P = 255$, and $M'_R = 789$. Reapportion.

(c) What's the "paradox" here?

(2) (a) Next suppose there are $m = 10$ seats on the governing council, and the districts in the town (districts B , P , and R) have populations $M_B = 54$, $M_P = 243$, and $M_R = 703$. Apportion the seats by Hamilton's method.

(b) In the previous problem, you already worked out how this changes when the number of seats goes up to $m = 11$. What's the "paradox" here?

(3) (a) Back to the original scenario ($m = 11$, $M_B = 54$, $M_P = 243$, $M_R = 703$): suppose that a new neighborhood is annexed to the town, with population $M_J = 580$. Would it be fair to give this new district $m_j = 6$ seats, increasing the size of the council to $m = 17$?

(b) Reapportion with $m = 17$. What's the paradox here?

HISTORICAL NOTE: A Brief History of Apportionment in the United States

The apportionment of the U.S. House of Representatives has an intriguing and convoluted history. As mandated by the United States Constitution, apportionments of the House are to take place every 10 years, following each census of the population. The real problem was that the Constitution left the method of apportionment and the number of seats to be apportioned essentially up to Congress. The only two restrictions stated in the constitution were that (1) "each State shall have at least one Representative," and (2) "The number of Representatives shall not exceed one for every thirty thousand" (Article I, Section 2).

Following the 1790 Census, and after considerable and heated debate, Congress passed the first "act of apportionment" in 1792. The bill, sponsored by Alexander

Hamilton (then Secretary of the Treasury), established a House of Representatives with $M = 120$ seats and apportioned under the method we now call *Hamilton's method*. In April of 1792, at the urging of then Secretary of State Thomas Jefferson, President George Washington vetoed the bill. (This was the first presidential veto in U.S. history.) Jefferson convinced Washington to support a different apportionment bill, based on a House of Representatives with $M = 105$ seats apportioned under the method we now call *Jefferson's method*. Unable to override the president's veto and facing a damaging political stalemate, Congress finally adopted Jefferson's bill. This is how the first House of Representatives came to be constituted.

Jefferson's method remained in use for five decades, up to and including the apportionment of 1832. The great controversy during the 1832 apportionment debate centered on New York's apportionment. Under Jefferson's method, New York would get 40 seats even though its *standard quota* was only 38.59. This apportionment exposed, in a very dramatic way, the critical weakness of Jefferson's method—it produces *upper-quota violations*. Two alternative apportionment bills were considered during the 1832 apportionment debate, one proposed by John Quincy Adams that would apportion the House using the method we now call *Adams's method* and a second one, sponsored by Daniel Webster, that would do the same using the method we now call *Webster's method*. Both of these proposals were defeated and the original apportionment bill passed, but the 1832 apportionment was the last gasp for Jefferson's method.

Webster's method was adopted for the 1842 apportionment, but in 1852 Congress passed a law making Hamilton's method the "official" apportionment method for the House of Representatives. Since it is not unusual for Hamilton's method and Webster's method to produce exactly the same apportionment, an "unofficial" compromise was also adopted in 1852: Choose the number of seats in the House so that the apportionment is the same under either method. This was done again with the apportionment bills of 1852 and 1862.

In 1872, as a result of a power grab among states, an apportionment bill was passed that can only be described as a total mess—it was based on no particular method and produced an apportionment that was inconsistent with both Hamilton's method and Webster's method. The apportionment of 1872 was in violation of both the constitution (which requires that some method be used) and the 1852 law (which designated Hamilton's method as the method of choice).

In 1876 Rutherford B. Hayes defeated Samuel L. Tilden in one of the most controversial and disputed presidential elections in U.S. history. Hayes won in the Electoral College (despite having lost the popular vote) after Congress awarded him the disputed electoral votes from three southern states—Florida, Louisiana, and South Carolina. One of the many dark sidebars of the 1876 election was that, had the House of Representatives been legally apportioned, Tilden would have been the clear-cut winner in the Electoral College.

In 1882 the *Alabama paradox* first surfaced. In looking at possible apportionments for different House sizes, it was discovered that for $M = 299$ Alabama would get 8 seats, but if the House size were increased to $M = 300$, then Alabama's apportionment would decrease to 7 seats. So how did Congress deal with this disturbing discovery? It essentially glossed it over, choosing a House with $M = 325$ seats, a number for which Hamilton's method and Webster's method would give the same apportionment. The same strategy was adopted in the apportionment bill of 1892.

In 1901 the *Alabama paradox* finally caught up with Congress. When the Census Bureau presented to Congress tables showing the possible apportionments under Hamilton's method for all House sizes between 350 and 400 seats, it was pointed out that two states—Maine and Colorado—were impacted by the Alabama paradox: For most values of M starting with $M = 350$, Maine would get 4 seats, but for $M = 357, 382, 386, 389,$ and 390 , Maine's apportionment would go down to 3 seats. Colorado would get 3 seats for all possible values of M except $M = 357$, for which it would only get 2 seats. For $M = 357$, both Maine and Colorado lose, and, coincidentally, this just happened to be the House size that was proposed for the 1901 bill. Faster than you can say "we are being robbed," the debate in Congress escalated into a frenzy of name-calling and accusations, with the end result being that the bill was defeated and Hamilton's method was scratched for good. The final apportionment of 1901 used Webster's method and a House with $M = 386$ seats.

Webster's method remained in use for the apportionments of 1901, 1911, and 1931. (No apportionment bill was passed following the 1920 Census, in direct violation of the Constitution.)

In 1941 Congress passed a law that established a fixed size for the House of Representatives (435 seats) and a permanent method of apportionment known as the *method of equal proportions*, or *Huntington-Hill method*. (For a discussion of the Huntington-Hill method see Mini-Excursion 1.) The 1941 law (*Public Law 291, H.R. 2665, 55 Stat 761: An Act to Provide for Apportioning Representatives in Congress among the Several States by the Equal Proportions Method*) represented a realization by Congress that politics should be taken out of the apportionment debate and that the apportionment of the House of Representatives should be purely a mathematical issue.

Are apportionment controversies then over? Not a chance. With a fixed-size House, one state's gain has to be another state's loss. In the 1990 apportionment Montana was facing the prospect of losing one of its two seats—seats it had held in the House for 80 years. Not liking the message, Montana tried to kill the messenger. In 1991 Montana filed a lawsuit in federal District Court (*Montana v. United States Department of Commerce*) in which it argued that the Huntington-Hill method is unconstitutional and that either *Adams's method* or *Dean's method* (see Project A) should be used. (Under either of these methods, Montana would keep its two seats.) A panel of three federal judges ruled by a 2-to-1 vote in favor of Montana. The case then went on appeal to the Supreme Court, which overturned the decision of the lower federal court and upheld the constitutionality of the Huntington-Hill method. But it is just a matter of time before the issue will surface again. As long as there are winners and losers in the apportionment game, the debate over what is the "right" apportionment method is not going to go away. ■