MATH 19-01: HW 7

(1) Show that Dictatorship is a Pareto-efficient and strongly monotonic single-winner system. (In other words, it satisfies the hypotheses of the Müller-Satterthwaite theorem.) Note there are three separate things to verify here.

(2) *This is an opinion question: your answer can be anything as long as you explain your reasoning.* How reasonable is it to insist that a voting system be single-winner? Does your answer change if the number of candidates (n) is high or low? Does it change if the number of voters (N) is high or low?

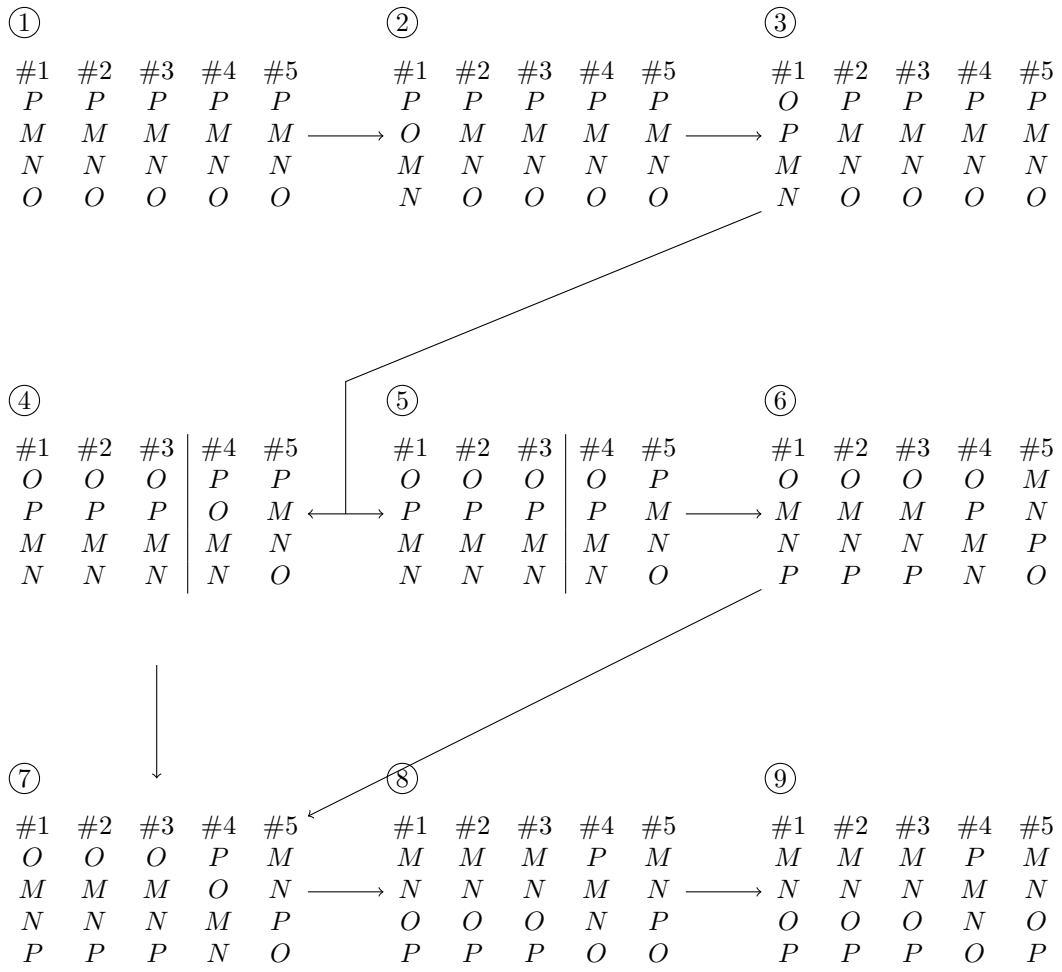
(3) Consider the following preference schedules:

×3	×2	×2
X	Y	Z
Z	X	Y
Y	Z	X

×3	×2	×2
X	Y	Y
Z	X	Z
Y	Z	X

Who wins each one, by the beatpath method? Considering those answers, does that tell you whether beatpath is strongly monotonic?

(4) (a) Following the proof of Müller-Satterthwaite, suppose you know you're working with an unknown single-winner voting system that is Pareto-efficient and strongly monotonic. Narrate the proof using the following sequence of detailed preference schedules.



(b) If we're showing that the unknown voting system is Dictatorship of the kth voter using this proof technique, what is the value of k in the example above?

(c) The point of getting to a "pathological" preference schedule like ⑨ is that from there you can get to ANY detailed schedule in which voter k ranks candidate P first with a combination of moves neutral to and favorable to P.

Check this by filling in a preference schedule in the middle where the transitions are as described here.

⑨

#1	#2	#3	#4	#5
M	M	M	P	M
N	N	N	M	N
O	O	O	N	O
P	P	P	O	P

$\xrightarrow[\text{to } P]{\text{neut}}$

⑩

#1	#2	#3	#4	#5

$\xrightarrow[\text{to } P]{\text{fav}}$

⑪

#1	#2	#3	#4	#5
P	O	N	P	O
O	P	O	O	P
M	M	M	N	M
N	N	P	M	N

Who wins in that final schedule and why?

(d) Note that ⑪ could have been *anything at all* as long as voter k liked P best! Explain why this finally proves that voter k is a "Dictator."