MATH 19-01: HW 11

Elementary counting and probability

All answers require brief explanations and not just numbers or formulas.

(1) Consider a standard American deck of cards, with 52 cards in four suits ($\heartsuit, \spadesuit, \clubsuit, \diamondsuit$) with thirteen different values ($A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K$).

(a) How many different pairs are there? (a *pair* is two cards of the same value, for instance $8\heartsuit/8\spadesuit$)

(b) How many ways to draw two cards from the deck?

(c) What is the probability that if you draw two cards you get a pair?

(d) How many different flushes are there? (a *flush* is five cards of the same suit)

(e) How many different ways to draw five cards from the deck?

(f) What is the probability that if you draw five cards you get a flush?

(2) Consider flipping a fair coin 20 times.
    (a) How many ways can this turn out?

    (b) What is more likely, getting equal numbers of heads and tails or getting ≥ 18 of one kind?

(3) Seven Democrats and nine Republicans are running for a university's Board of Regents. The rules say that there should be four Ds and four Rs on the board. How many choices are possible?

(4) A state's presidential primary runs as follows: Each voter must pick a party, and then rank the candidates from first to last within that party. Suppose there are three Democrats and five Republicans in the race. How many possible ballots can you cast?

(5) Suppose two different voters are ranking three candidates. If all possibilities are equally likely, what is the probability that their votes are the same?

(6) Suppose ten different voters are ranking three candidates (A, B, C) and all possible rankings are equally likely.

(a) How many rankings are there with A in first place?

(b) What is the probability that everyone ranks A first?

(c) How many rankings are there with A preferred to B?

(d) What is the probability that A is unanimously preferred to B?

(e) What is the probability that $\mathcal{W}_{\text{plur}} = \{A, B\}$?